19th Meditarranean mathematical olympiad

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Problem 1

Determine all integers $n \geq 1$ for which the number $n^8+n^6+n^4+4$ is prime.

Problem 2

Let ABC be a triangle . D is the foot of the internal bisector of the angle A. The perpendicular from D to the tangent AT (T belong to BC) to the circumscribed circle of ABC intersect the altitude AH_a at the point I (H_a belong to BC).

If P is the midpoint of AB and O is the circumcircle, TI intersect AB at M and PT intersect AD at F, prove that MF is perpendicular to AO.

Problem 3

Let a,b,c be positive real numbers such that $a+b+c=3$. Prove that

$$\sqrt{\frac{b}{a^2+3}}+\sqrt{\frac{c}{b^2+3}}+\sqrt{\frac{a}{c^2+3}} \leq \frac{3}{2}\sqrt[4]{\frac{1}{abc}}.$$

Problem 4

Consider a 25×25 chessboard with cells $C(i,j)$ for $1 \leq i,j \leq 25$. Find the smallest possible number n of colors with these cells can be colored subject to the following condition: For $1 \leq i < j \leq 25$ and for $1 \leq s < t \leq 25$, the three cells $C(i,s), C(j,s), C(j,t)$ carry at least two different colors.