Junior Balkan MO 1998

Athens, Greece

$\boxed{1}$ Prove that the number $\underbrace{111\ldots11}_{1997}\underbrace{22\ldots22}_{1998}5$ (which has 1997 of 1-s and 1998 of 2-s) is a perfect square.

Yugoslavia

$\boxed{2}$ Let $ABCDE$ be a convex pentagon such that $AB = AE = CD = 1$, $\angle ABC = \angle DEA = 90^\circ$ and $BC + DE = 1$. Compute the area of the pentagon.

Greece

$\boxed{3}$ Find all pairs of positive integers (x, y) such that

$$x^y = y^{x-y}.$$

Albania

$\boxed{4}$ Do there exist 16 three digit numbers, using only three different digits in all, so that the all numbers give different residues when divided by 16?

Bulgaria