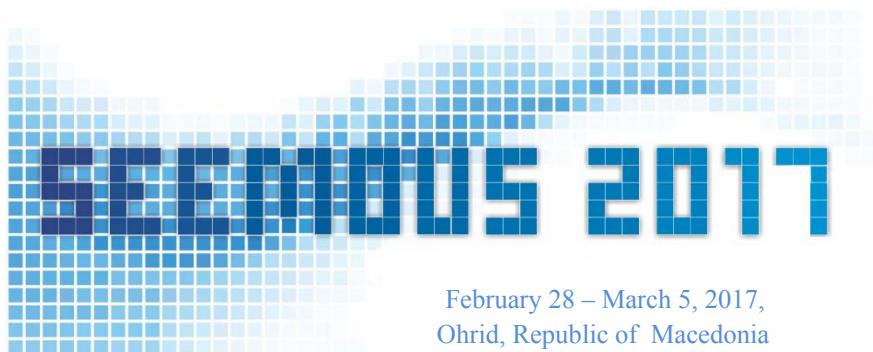




Problems

Problem 1. Goý, $A \in M_2(\mathbb{R})$ üçin

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

şert ýerine ýetýän bolsa,

$$a^2 + b^2 + c^2 + d^2 < \frac{1}{5}.$$

$I + A$ ters matrissasynyň bardygyny subut ediň.

Problem 2. Goý $A, B \in M_n(\mathbb{R})$.

a) Islendik $\varepsilon \in (-a, a)$, $\varepsilon \neq 0$ san üçin, matriks deňlemäniň

$$AX + \varepsilon X = B, \quad X \in M_n(\mathbb{R}),$$

ýeketäk çözüwi bar bolar ýaly $X(\varepsilon) \in M_n(\mathbb{R})$ $a > 0$ bardygyny subut ediň.

b) Eger $B^2 = I_n$ bolsa, subut ediň

$$\lim_{\varepsilon \rightarrow 0} \varepsilon \cdot \text{Tr}(BX(\varepsilon)) = n - \text{rank } A.$$

Problem 3. Goý $f: \mathbb{R} \rightarrow \mathbb{R}$ üznüksiz funksiýa bolsun. Subut ediň

$$\int_0^4 f(x(x-3)^2) dx = 2 \int_1^3 f(x(x-3)^2) dx.$$

Problem 4. a) Goý $n \geq 0$ bitin san bolsun. Hasaplaň $\int_0^1 (1-t)^n e^t dt$.

b) Goý $k \geq 0$ fiksirlenen bitin san we goý $(x_n)_{n \geq k}$ yzygiderlik aşakdaky ýaly berlen bolsa

$$x_n = \sum_{i=k}^n \binom{i}{k} \left(e - 1 - \frac{1}{1!} - \frac{1}{2!} - \dots - \frac{1}{i!} \right).$$

zygygiderligiň ýygnaýandygy subut ediň we predelini tapyň.

Time for work 5 hours