

Junior Balkan MO 2007

Shumen, Bulgaria

1. Let a be positive real number such that $a^3 = 6(a+1)$. Prove that the equation $x^2+ax+a^2-6 = 0$ has no real solution.
2. Let $ABCD$ be a convex quadrilateral with $\angle DAC = \angle BDC = 36^\circ$, $\angle CBD = 18^\circ$ and $\angle BAC = 72^\circ$. The diagonals and intersect at point P . Determine the measure of $\angle APD$.
3. Given are 50 points in the plane, no three of them belonging to a same line. Each of these points is colored using one of four given colors. Prove that there is a color and at least 130 scalene triangles with vertices of that color.
4. Prove that if p is a prime number, then $7p + 3^p - 4$ is not a perfect square.