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WORKING WITH MATHEMATICALLY GIFTED STUDENTS IN PRIMARY EDUCATION – PART TWO

UDC: 373.3.011.3-056.317:51

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Abstract. Most commonly, the differentiation of mathematics instruction is realized by working with mathematically gifted students. Traditionally this is as a part of mathematical sections, by preparing the students for mathematical competitions. However, we consider that it is going to be much more efficient if it is implemented continuously and following a specially prepared program for these students. For this purpose in this paper, we present the integral teaching program for working with gifted students in Grade VII (Grade VI) in the nine-year long (eight-year long) primary education. This teaching program is an upgrade of the corresponding teaching programs for the given grades, developed by the same authors. Alongside, almost the complete teaching program is supplemented by a system of tasks for particular topics (solved and unsolved), given in [10] – [19].

1. INTRODUCTION

Most often, studying creativity, giftedness and talent comes down to defining them and creating instruments for recognizing and identifying talented and gifted children. In this paper, we are not going to analyze these questions, because they are already analyzed in the existing literature, such as in [1], [2], and [20]. Additionally, the gifted students and their education, i.e. the methods and forms of work with the gifted students are another important aspect in the work with these students. The existing literature also analyzes these questions, and integral programs for work with gifted students in mathematics, aged 7 -11, are presented in [3] –[5].

Systems of tasks from different areas are also present in the previously mentioned works and they can serve for work with the mathematically gifted students aged 7 to 11. These systems of tasks are compiled in the frames of the European MATHEU project, in which they are overambitiously called didactic pillars. We can say that the use of particular systems of tasks when working with the mathematically gifted students is used by several authors many years prior to the realization of the MATHEU project. For example, such a system of tasks is presented in [6], which is smaller in volume and it is intended for acquisition of the scientific methods by the mathematically gifted students.

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Key words and phrases. mathematically gifted students, primary education

In the following analysis, we are going to present an integral teaching program for work with mathematically gifted students aged 12-13, and for the topic Theory of numbers, we are going to present a system of tasks, which we believe will serve when working with the mathematically gifted students.

2. PROGRAM FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 12-13

Integral teaching programs when working with mathematically gifted students in the initial education, i.e. ages 7 -11 are presented in [3] – [5]. This part will also cover it in terms of work with students aged 12-13, i.e. with students from seventh (sixth) grade in the nine-year(eight) long primary education. In the preparation of this program, as well as in the preparation of the previous programs, we implemented experiences from the work with mathematically gifted students from several countries, the ones coming from the Russian Federation, Bulgaria, Romania, Serbia, Croatia, Bosnia and Herzegovina and Macedonia being the most prominent.

The goals of the teaching program for students aged 12-13 are the following:

- The students understand the term fraction, do the operations with fractions and use them when solving tasks,
- The students represent values in terms of percentages and use percent operations,
- The students understand the need for the introduction of negative numbers and generating the set of integers, as well as to acquire the term absolute value and learn the arithmetic operations in the set of integers,
- The students understand the construction of the set of rational numbers and to learn the arithmetic operations and their properties in the set of rational numbers,
- The students solve equations with one variable and use them when solving a word problem tasks,
- The students understand the term mapping, and use axial and central symmetry to map figures,
- The students distinct axial and central symmetric figures as well as to determine the axis of symmetry and center of symmetry of figures
- The students understand the characteristics of the triangle and quadrangle, their properties and classification,
- The students calculate the perimeter of triangles and quadrangles,
- The students understand the relation of congruent triangles and use the attribute of congruence in simple tasks,
- The students understand the need for proving a theorem and are able to prove some theorems,
- The students are able to solve logical tasks,

- The students learn the basic combinatory principles and combinatory configurations in an informal way,
- The students are able to solve tasks with coloring, covering and cutting figures into simpler figures, and creating a new figure from the cut pieces,
- The students use the method of invariants (in an informal way),
- Develop the qualities of thinking, such as: elasticity, pattern-making, depth, rationalization, and critical thinking,
- The students learn in an informal way the scientific methods: observation, comparison, experiment, analysis, synthesis, and the axiomatic method
- The students learn in an informal way the types of conclusion-making: induction, deduction, and analogy, while presenting suitable examples from which the students will learn that analogy based conclusion is not always correct.

The following content needs to be learned to achieve the previously mentioned goals:

Topic I. *Integers and rational numbers*: direction, positive and negative numbers, additive inverses, set of integers, absolute value and comparing integers, fractions, types of fractions, extending and reducing fractions, creating common denominators for the fractions and comparing fractions, arithmetic operations with fractions (additions, subtraction, multiplication and division), complex fractions, order of the arithmetic operations and calculating the value of a numerical expression, percentages: concept for percent and percent value, transforming a decimal number into percent, as well as a percent in the form of a fraction and a decimal number, the set of rational numbers, absolute value of a rational number and comparing rational numbers, arithmetic operations with rational numbers (addition, subtraction, multiplication and division) and their properties, calculating the value of a numerical expression with rational numbers, solving linear equations containing rational numbers, exponentiation with a natural number as an exponent, operations with exponents.

Topic II. *Theory of numbers*: general and specific indicators for divisiveness (divisiveness with 7, 8, and 11), numeric systems, greatest common divisor, Euclid's algorithm and lowest common multiplier, prime and composite numbers, Eratosthenes sieve, infinity of the set of prime numbers, basic arithmetic theory, elementary Diophantine equations.

Topic III. *Word problem tasks*: tasks with numbers and numerals, tasks with measure numbers, tasks with percentages, and tasks involving money.

Topic IV. *Geometry*: mapping, definition and basic properties, axial symmetry, definition and basic properties, mapping figures by axial symmetry, axisymmetric figures, line segment bisection, angle bisection and its properties, perpendicular lines, distance from point to plane, central symmetrical figures, triangle: elements of a triangle and types of a triangle, altitude of a triangle and orthocenter of a triangle, median of a triangle and a triangle centre, bisection of a side of a triangle and a center of a excircle of a triangle, bisection of an angle of a triangle and a center of an incircle of a triangle, circumscribed circles of a triangle, tangent of a circle, congruent figures, congruent triangles, indicators for congruent triangles: the indicator side-angle-side (SAS), the indicator angle-side-angle (ASA) and the indicator side-side-side(SSS), properties of an isosceles triangle, parallel lines, parallel postulate, transversal of parallel lines, angles of a transversal, angles with parallel sides and angles with normal sides, sum of interior angles of a triangles and sum of exterior angles of a triangle, median of a triangle, relation between sides and angles of a triangle, basic constructive tasks, construction of a tangent of a circle and construction of a triangle with given elements, elements of a quadrangular, sum of angles in a quadrangular, quadrangular types, parallelogram, parallelogram properties, rhombus and square, trapezoid, elements of a trapezoid and its properties, isosceles trapezoid and kite, construction of a quadrangular with given elements, perimeter of a quadrangular: parallelogram, trapezoid and kite, concept of area, area of a triangle and a quadrangular, concept for volume, volume of a square and a cuboid.

Topic V. ***Sets, logic and combinatorics*****:** sets, number of elements of a set, operations with a set, Venn diagram, determining a set in given conditions, logical tasks, games and strategies, Dirichlet's principle (intuitive use), counting and recounting by using the principles of sum, difference and product (intuitive use), coloring, covering and cutting, elementary games and strategies.

3. AN EXAMPLE OF A SYSTEM OF TASKS FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 12-13

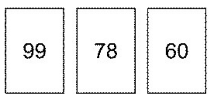
Similarly, as the programs presented in [3], [4] and [5], for the realization of the suggested program for working with mathematically gifted students aged 12-13, we must prepare adequate teaching aids, i.e. textbooks that will be mandatorily supplemented by adequate collections of tasks. Further on, we are going to give an example of a system of tasks for this age group on the topic of the Theory of numbers.

Task 1. What is the sum of all five-digit numbers created with the digits 1,2,3,4 and 5, each digit occurring exactly once?

Task 2. Between the digits of any two-digit number written with the same digits insert two zeroes. The new number is exactly 91 times greater than the initial number. Prove!

Task 3. Two numbers act like 19 : 8. If the sum of these numbers is divided by their difference, we get a quotient of 2 and a remainder of 20. What are these numbers?

Task 4. Let a,b,c,d be different digits and each of them be a prime number. Write all $\overline{ab10cd}$ divisible by 264.



Task 5. On 3 cards, Marko wrote 6 different numbers, one on each side of the cards. The sum of the two

numbers written on each of the cards is equal. Marko set the cards in such a way that only three numbers can be seen, as on the image on the right. We know that the hidden numbers are prime numbers. Find the hidden numbers.

Task 6. Determine a two-digit number such that the sum of the number and the number written with the same digits in a reverse order is a square of a natural number.

Task 7. Let T be a three-digit number. A new three-digit number is formed by switching the position of the last two digits. Their sum is a four-digit number which begins with 195. Determine all such three-digit numbers.

Task 8. The first row of a table with three columns contains the numbers a, b and c. The second row below contains the numbers $a_1 = a - b,\ b_1 = b - c$ and $c_1 = c - a$. Analogously, the third row contains $a_2 = a_1 - b_1, b_2 = b_1 - c_1$ and $c_2 = c_1 - a_1$, etc. Find the greatest possible number in the order in which the number 2013 can occur in an adequate choice of the starting numbers.

Task 9. Determine the four-digit number $\overline{xyzt}$ for which the following is satisfied:

$$\overline{xyzt} + 4 \cdot \overline{yzt} + 2 \cdot \overline{zt} = 2018.$$

Task 10. Instead of a and b use such digits so that the number $\overline{4a1b}$ is divisible by 12.

Task 11. When dividing the number $n + 125$ with the number 19 there is a remainder of 7. Calculate the remainder when dividing the number n by 19.

Task 12. Find the numbers x, y so that the number $\overline{x74y}$ is divisible by 15.

Task 13. Determine the digits a and b so that the number $\overline{a783b}$ is divisible by 56.

Task 14. Let $S(n)$ be sum of the digits of the natural number n. Is there exist a natural number for which the following applies?

a) $S(k) + S(k^2) = 2008$,

b) $S(k) + S(k^2) = 2009$.

Task 15. The product of two natural numbers is 384, and their lowest common multiplier is 48. Determine the numbers.

Task 16. Find the greatest four-digit number for which when divided by 3, 4, 5, 6, and 7, the remainder is 2.

Task 17. Determine all three-digits natural numbers for which when divided by 7 the remainder is 2, when divided by 9 the remainder is 4, and when divided by 12 the remainder is 7.

Task 18. The number 333 express as a product of two numbers so that each of the multipliers is smaller than 10.

Task 19. The difference between a two-digit number and the number written with the same digits but in reverse order is 45. The sum of these numbers is an exact square of a natural number. List all two-digit numbers that have these properties.

Task 20. Determine the integers a,b and the prime number p so that $|ab|p=4022$.

Task 21. The product of two three-digit numbers is written with only several threes. What are the numbers?

Task 22. If p is a prime number, prove that

a) p^3+1987 is a composite number,

b) $p^{1987}+1987$ is a composite number.

Task 23. Calculate all natural numbers n so that the numbers $3n-4$, $4n-5$ and $5n-3$ are prime numbers.

Task 24. Calculate all prime numbers p for which the following inequations are true: $\frac{3}{16}<\frac{5}{p}<\frac{2}{7}$.

Task 25. Calculate all prime numbers p so that

$$\frac{665}{1993}<\frac{5}{p}<\frac{997}{1994}.$$

Task 26. Calculate all prime numbers p,q and r so that

$$2p+3q+4r=2022.$$

Task 27. A natural number n, when divided by 3, has a remainder a, when divided by 6, it has a remainder b, and when divided by 9, it has a remainder c. We know that $a+b+c=15$. Calculate the remainder, when the number n is divided by 18.

Task 28. What are the natural numbers a and b for which the following applies

$$\text{NZD}(a,b)=8 \text{ and } \text{NZS}(a,b)=168.$$

Task 29. Are there exist natural numbers x and y so that

$$\text{NZD}(x,y)+\text{NZD}(x+1,y+1)=x-y\,?$$

Task 30. The houses from the left side of the street are numerated with odd numbers, and the houses on the right side of the street are numerated with even numbers. The sum of all house numbers on one side is 1309, and on the other is 2162. How many houses are there on this street?

Task 31. Calculate all integers n for which $\frac{n+4}{3n-2}$ is an integer.

Task 32. Calculate all pairs of integers (a,b) for which the following applies $a=\frac{4b-5}{b-2}$.

Task 33. Solve the equation in the set of integers.

$$10xy+16x+5y=2006.$$

Task 34. Solve the equation in the set of natural numbers

$$\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=1.$$

Task 35. Determine all natural numbers a,b,c,d,e so that $2<a<b<c<d<e$ and

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}=1.$$

Task 36. Calculate the three-digit number that is divisible by 9, whose digit for tens is for 4 smaller than the digit of the ones, and the product of its digits is equal to 0.

Task 37. Let n be a natural number greater than 1. Prove that the value of the fraction $\frac{10^n+8}{36}$ is an integer.

Task 38. Determine the smallest natural number which when multiplied by 2 becomes the square of a natural number, and when multiplied by 3 becomes the cub of another natural number.

Task 39. Write the number 313224_4 in base 10 system.

Task 40. Write the number $5a19_{11}$ in base 10 system.

Task 41. Find a natural number b so that in a b base system, the number 792 is divisible by 297.

Task 42. Write the tables for addition and subtraction of nonnegative integers smaller than the base of the base 8 numeral system.

4. CONCLUSIONS

In the previous analysis, we discussed only the work with gifted students in math in seventh (sixth) grade in the nine-year long (eight-year long) education. We need to emphasize that the work with the gifted students in math should be part of the differentiation of the instruction, and:

- The work with the gifted students, especially the students gifted in math, should not only be a declarative effort of the responsible institutions, which according to the practice thus far consider that for its realization it is sufficient to give accreditations for organizing competitions and say that it should be done by the teacher in the programs,
- A special teaching program should be developed for each age group, such as the program that is contained in this paper, and it needs to be carried out in the course of the whole school year, not right before the math competitions, which is the case in the current practice.
- Adequate didactic materials will be prepared for the realization of the program for work with gifted students, supported by the respon-

sible institutions, which as far as we know is not the case in any country of our immediate surroundings.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

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INVESTIGATING SOME ASPECTS OF PRE-SERVICE PRIMARY SCHOOL TEACHERS' MATHEMATICS ANXIETY

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Abstract. One of the challenges that mathematics teacher educators face is the fact that teachers' own schooling experiences shape their beliefs about teaching and how they interact with pupils. Research suggest that pre-service primary school teachers experience higher level of mathematics anxiety than other university students. On the other hand many studies point out that higher levels of mathematics anxiety in teachers might affect their instructional practices, the willingness to embrace innovations and are related to mathematics anxiety and lower achievement of their pupils. This paper aims to investigate if there are any indications of mathematics anxiety in first and second year teacher students enrolled in the study year of 2019/2020 in two teacher education programs at the Faculty of Education in Jagodina, University of Kragujevac, Serbia. The sample consisted of 95 teacher students and a quantitative research method was applied. Results have implications for possible improvements of teacher education programs, and can be used as support to encourage further investigations on mathematics anxiety of future primary school and preschool teachers and ways of reducing it.

1. INTRODUCTION

In spite of its role, importance and broad application in the development of world and modern society, mathematics is still considered as a difficult school subject. Research point out that many students encounter learning difficulties and have poor performance in mathematics [1]. As Peker indicates, there are various factors that might affect students learning abilities such as instruction, teacher beliefs, lack of self-confidence, mathematics anxiety, etc [14]. It is therefore important to equip future teachers of mathematics with adequate teaching competences and skills which will enable them to respond to the fast changing needs of learners in mathematics classrooms. One of the challenges that mathematics teacher educators face is the fact that teachers' own schooling experiences shape their beliefs about teaching and how they interact with pupils. As a result of their classroom experiences, the majority of pre-service primary teachers come to teacher education courses with deeply rooted anxieties and attitudes about mathematics [6]. One of the ongoing concerns for pre-service teacher education programs is the mathematics anxiety [12].

Key words and phrases. Mathematics anxiety, pre-service primary school teachers, teaching mathematics, teacher education programs.

Richardson and Suinn define mathematics anxiety as the "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" ([17], p. 551). Trujillo & Hadfield defines it as a state of discomfort which occurs as a response to situations that involve mathematical tasks and that are perceived as a threat [21]. Mathematics anxiety has important implications for teacher students' learning and might inhibit performance in mathematics courses and affect future opportunities for engagement in mathematics [16]. Research indicate that pre-service primary school teachers experience higher level of mathematics anxiety than other university students ([4], [18], [21]). The mathematics anxiety of pre-service primary teachers might be attributed to their prior mathematics experiences at school level ([3], [22]) and lack of mathematical knowledge ([8], [10]). A significant body of literature exists suggesting that higher levels of mathematics anxiety in teachers might affect their instructional practices, less-skilled teaching, the willingness to embrace innovations and are related to mathematics anxiety and lower achievement of their pupils ([5], [7], [11], [12], [19], [20], [21], [23], [24]). Mathematics anxiety affects teachers' behaviour since it impacts their confidence in abilities to do and understand mathematics [8]. Teachers with higher level of mathematics anxiety tend to use more traditional methods, direct instruction and less student-centered approaches ([3], [7]), and they spent less time in mathematics lesson preparation, and do not effectively use instructional time [5]. Ramirez et al. point out that mathematically anxious teachers tend to "harm student mathematics learning by responding angrily when students request help with mathematics" and spend less time to respond to students' question when compared to less anxious teachers [15]. The same authors also emphasize that these teachers primarily promote algorithmic thinking and problems with a single solution/problem solving method. Olson and Stoehr indicate that "teachers who fell uncomfortable with mathematics may model their anxiety to students" ([12], pp. 73). Consequently, it leads to the transmission of anxiety and continuity of the mathematics anxiety phenomenon.
Mathematics anxiety may inhibit pre-service teachers from learning mathematical contents and developing instructional skills. Pre-service teachers with high level of mathematics anxiety are more likely to express negative attitudes towards mathematics and necessary mathematics university courses, and to have lower performance in mathematics teaching methodology courses ([3], [16]). In order to be able to make sense of students' mathematical thinking, pre-service teachers must develop strong mathematics content knowledge and rich conceptual understanding of mathematical pedagogy ([12], [25]). There is one more issue that must not be neglected. Researchers determined that female pre-service teachers demonstrated higher levels of mathematics anxiety than male [26]. Some of this differences might be related with the impact that female teachers' mathematics beliefs have on their female students' perceptions of their mathematical abilities [27]. These and the fact that majority of pre-service

primary teachers at teacher education faculties in Serbia are female, indicate that it is of extreme importance to pay attention to these aspect of educating future primary teachers. Since pre-service primary teachers represent the first face of mathematics to young children, it is of extreme importance to educate the future generations of teachers to be as effective as it is possible in teaching mathematics ([9], [13]).

Given that numerous studies show that teachers' mathematics anxiety has significant influence not just on their teaching practice, but on students' mathematics learning, achievement, attitudes and anxiety, we have decided to examine the pre-service primary teachers' level of mathematics anxiety at Faculty of Education in Jagodina, University of Kragujevac. The value of this study can be recognized in the fact that this is the first time that a research on pre-service primary teachers' mathematics anxiety had been conducted in Serbia. Pavlin-Bernardić, Vlahović-Štetić and Mišurac Zorica examined mathematics anxiety in pre-service primary teachers and pre-service mathematics teachers in Croatia [13]. The authors determined that although none of the groups of pre-service teachers indicated extremely high levels of mathematics anxiety, future primary teachers expressed higher level of mathematics anxiety than future mathematics teachers.

2. RESEARCH METHODOLOGY

This study aims to investigate if there were any indications of mathematics anxiety in first and second year teacher students enrolled in the study year of 2019/2020 in teacher education program at the Faculty of Education in Jagodina, University of Kragujevac, Serbia. The aim was realized through the following *research questions*:

1) to examine the mathematics anxiety level of pre-service primary and preschool teachers in general;

2) To examine the level of certain aspects of mathematics anxiety (oral and written examination, some situations in mathematics classes, use of mathematics in everyday life) of pre-service primary and preschool teachers;

3) to investigate if there are statistically significant differences in level of mathematics anxiety between pre-service primary and pre-service preschool teachers;

4) to determine if there are statistically significant differences in level of mathematics anxiety in regard to the year of study and average high school mathematics grades.

The study was conducted at the Faculty of Education in Jagodina, University of Kragujevac, Serbia. Since survey instruments were administered and numerical data collected, a quantitative method was used in analyzing the data. Data were collected through questionnaires. The participants were pre-service teachers (PST) enrolled in Preschool and Primary Teachers Education Programs (Year 1 and Year 2). All students were enrolled in Mathematics courses at Year

1/Year 2. Participants had not yet attended methodology of teaching Mathematics courses since these courses are taught at Year 3 and Year 4.

Sample. The research sample involved 95 teacher students. The study was conducted at the end of the summer semester of academic year 2019/2020. All PST participated in the study on voluntary basis. The sample distribution in regard to the year of study is presented in Table 1, in regard to the teacher education program in Table 2 and in regard to the average mathematics grade in high school in Table 3.

Table 1. Sample distribution in regard to the year of study

Year 1	*Year 2*
64	31

Table 2. Sample distribution in regard to the teacher education program

Preschool Education Program	*Primary Teachers Education Program*
44	51

Table 3. Sample distribution in regard to the average mathematics grade in high school

Average mathematics grade	*Year 1*	*Year 2*
2	10	1
3	27	7
4	12	10
5	15	13

Instrument. The used instrument was a questionnaire that consisted of two parts. In the first part, background information about PST was collected (mathematics grades, teacher education program and a year of study). The second part of the instrument contained Mathematics Anxiety Scale (MA) developed by Arambašić, Vlahović-Štetić and Severinac [2]. The MA scale consisted of 20 items. Each of the items was rated on a Likert scale (1 = not upset at all, 2 = slightly upset, 3 = very upset, 4 = extremely upset). The participants were asked to respond how upset they were in some situation that was related to mathematics or required them to use mathematics. In order to investigate the mathematics anxiety of PST, we used the MA scale for the reason that it was developed and used in a country (Croatia) with historically common educational background as that to Serbia. Therefore, no translation was needed, and we obtained authors' permission to use the scale in our research. The Cronbach's alpha reliability coefficient indicated acceptable reliability (α=0.948). We present some examples of research items in Table 4.

Table 4. Examples of some items of Mathematics Anxiety scale [2]

Example of items
When the next class is mathematics.
When I am having important mathematics test.
When I am having an oral examination in mathematics.
When we learn new mathematics contents.

The items were determined so that four subscales could be structured. The subscales examine the mathematics anxiety level of pre-service primary and preschool teachers in: oral examination (AS1); written examination (AS2), some situations in mathematics classes (AS3); use of mathematics in everyday life (AS4).

The statistical analyses were conducted using SPSS for Windows, version 20.0. For the purpose of statistical analysis, p values lower than 0.05 were considered statistically significant. The normality of data was evaluated with the use of the Shapiro-Wilk test of normality. For the quantitative analyses of data methods of descriptive statistics were used (frequency, percentage, mean, standard deviation, mean ranks), Welch ANOVA with Games-Howel post hoc for parametric variables and Kruskal-Wallis H test with Dunn post hoc for non-parametric variables. The effect size was estimated by using Cohen's d. The independent variables in the data analysis was the year of study.

3. RESULTS AND DISCUSSION

In general, pre-service and preschool teachers express small degree of anxiety towards mathematics (M = 2.31, SD = 0.71). Results showed that there were no statistically significant differences in level of anxiety towards mathematics in regard to the education program or year of study, for any item or group of items. According to that, the first part of discussion will be based on a global observation of all students' mathematics anxiety level.

The highest level of anxiety occurs in oral examination and in giving answers on mathematics classes (M = 2.94, SD = 0.98). The similar level of anxiety occurs whether an oral examination is announced (M = 2.95, SD = 0.99) or not (M = 2.93, SD = 1.12). This result is in favour with students' lower exam achievements, not just on those related to mathematics, where knowledge is tested by an oral examination.

Although students showed a slightly lower degree of anxiety according to written tests (M = 2.79, SD = 0.80), it is still very high and may be one of the reasons for lower student achievement. The lowest anxiety in students is when they know that they have to start learning for a written test (M = 2.40, SD = 0.95). As the term of written examinations approaches, it increases, so the day before the written examination it becomes high (M = 2.69, SD = 1.01), and it becomes even higher during the written examination itself (M = 2.86, SD = 1.01). However, although we expected that the anxiety would be lower after the control exercise, it is actually the highest in the period while waiting for the results of the written test (M = 2.99, SD = 0.98). It was noticed that the degree of anxiety in students with unannounced written tests (M = 2.99, SD = 1.11) is equal to the degree of anxiety they have while waiting for the results of these tests.

Unlike written and oral tests, a small degree of anxiety in students occurs in math classes (M = 2.12, SD = 0.80). Similar to written tests, the lowest level of anxiety occurs just before the start of math class (M = 1.83, SD = 1.06) and remains at a similar level in class during the process of working on a math problem on the board, either by another student (M = 1.87, SD = 0.95) either by teachers (M = 1.92, SD = 0.96). However, anxiety becomes greater with the knowledge that the processing of new material begins in class (M = 2.23, SD = 1.05) and remains unchanged even while the teacher interprets the material (M = 2.24, SD = 1.06). The anxiety that students feel is no less even in the moments when they encounter mathematical formulas in class (M = 2.34, SD = 0.99) or when solving mathematical problems that are not the direct application of learned patterns (M = 2.43, SD = 0.97), when is the anxiety on the verge of becoming very great.

We registered that the lowest level of mathematics anxiety is in students' use of mathematics in everyday life (M = 1.91, SD = 0.70). It is interesting that level of mathematics anxiety increases when student solving harder mathematical problems. Mathematical anxiety is almost absent while students are solving simple mathematical problems (M = 1.57, SD = 0.83), but it becomes almost high when they face with complex problems (M = 2.46, SD = 1.02). Students' attitude toward mathematical literature is also interesting. In first touch with mathematical literature, students have a slight anxiety (M = 1.61, SD = 0.89) and it increases during literature review (M = 2.09, SD = 1.10). The good thing is that it crashes again during long-term using of literature (M = 1.85, SD = 0.98).

As we have already said, the observed global characteristics are the same for all of students, no matter to the education program or a year of studying. However, a level of a students' high school knowledge was a key parameter to investigate mathematics anxiety. Level of high school knowledge was measured as a average high school mathematics grade. Although we wanted to take mathematics grade at the end of a fourth year of high school as a key parameter it was not possible because there were students who only had mathematics in first two years of high school.

Namely, it was determined that in the first year of study there is a statistically significant difference in the degree of anxiety in the three subscales (Table 5), where it is characteristic that the degree of anxiety is inversely proportional to the grade in mathematics in high school (Table 6). This result is in accordance with some foreign research that we had the opportunity to get acquainted with.

Table 5. Statistical significance of the degree of anxiety for each of the subscales in relation to the average grade from high school

	F	*p*
AS1	8.845	**.000**
AS2	3.335	**.025**
AS3	1.702	.176
AS4	2.925	**.041**

Table 6. Degrees of anxiety on each of the subscales in relation to the average grade from high school

Average highschool mathematics grades	*AS1 (M)*	*AS2 (M)*	*AS3 (M)*	*AS4 (M)*
2	3.65	3.16	2.46	2.30
3	3.35	3.01	2.32	2.12
4	2.46	2.65	2.10	1.96
5	2.20	2.32	1.79	1.53

In each group, we note that there is a higher level of anxiety in students with average grade two in mathematics in highschool, while the level of anxiety in students who had average grade five is reduced by an average of 31.76%. Also, it can be noticed that students with a lower grade in mathematics from highschool have a higher level of mathematics anxiety during oral examination, while those with a higher grade are more anxious during written exams. In all groups, the lowest level of anxiety and the smallest difference in groups occur when it comes to solving classic textual tasks in which two quantities are known, and based on them third quantity should be determined by applying one of the four arithmetic operations. The highest level of anxiety and the greatest difference occur in frontal oral examination in mathematics class. In 80% of cases students with a average grade two from highschool stated they have the highest, fourth, level of anxiety on this item while the remaining 20% defined for the third level of anxiety (they were very upset).

It is interesting to look at the answers of Year 2 students of the Primary Teachers Education Program. Due to the sample size, we cannot talk about the statistically significance of the results but based on the mean values of the answers we can notice that the level of anxiety of students with lower highschool grades is a little bit lower than that Year 1 students (Table 7). The level of anxiety in oral examination is still the highest among Year 2 students, but in relation to Year 1 students, the anxiety that occurs when waiting for the results of a written exam is more evident.

Table 7. Level of anxiety according to each of the subscales regard to the average highschool grade of the Year 2 students

Average highschool mathematics grades	*AS1 (M)*	*AS2 (M)*	*AS3 (M)*	*AS4 (M)*
3	3.14	2.97	2.31	2.26
4	2.85	2.86	2.09	1.75
5	2.85	2.63	1.88	1.57

At this point, we surely cannot draw any long-term conclusions in terms of whether and how a level of students' anxiety changes over a long period at the Faculty because long-term observation and scaling is required.

4. CONCLUSION

Pre-service primary and preschool teachers in general expressed small degree of anxiety towards mathematics. The impression is that if we want students to successfully master mathematical content and thus successfully work on their future vocation, we would have to pay attention to the different spectrum of factors that can enable us to do so.

For example, a higher level of anxiety during oral exams of students with lower mathematics grades indicates that students in addition to giving an incorrect answer, fear of public appearance creates additional discomfort. Thus, we should work on strengthening the self-confidence of this group of students for public presentations. Especially, if we have in mind their future vocation and the demands they will face with. In contrast, students with higher mathematics grades have shown that oral examination makes them less anxious and closer to them than giving answers on paper. The common denominator for all students is the uncertainty they express while waiting for the results of written exam, which indicates uncertainty in the results of their work.

Furthermore, keeping in mind that the level of anxiety is influenced by previous experiences of students, it is necessary to pay more attention to overcoming the resulting conditions when they come to the faculty. It is also necessary to prevent possible side effects of their faculty education, especially considering the fact that, in their future occupation, they will directly teach mathematics or develop children's initial mathematical concepts. Also, they will indirectly use the mathematics contents in almost a third of the contents that children will be learning.

It is of extreme importance that teacher education programs give pre-service primary teachers adequate training on mathematics content knowledge and pedagogical knowledge. Furthermore, teacher educators must be aware of the fact that teachers' negative attitudes and beliefs towards mathematics, and high levels of mathematics anxiety might affect their teaching behaviour, teaching process, choice of teaching methods and strategies, readiness to embrace innovation, but also pupils' mathematics achievement, attitudes, beliefs and mathematics anxiety. The studies show that "the teachers who are not specialist mathematics teachers at the start of their training follow the 'heritage' path" ([9], pp. 19), i.e. they teach as they have been taught. This means that teacher education programs must provide opportunities for future teachers to experience mathematics in the way they should teach. Since mathematics anxiety of future primary teachers might be attributed to their previous mathematics experiences and lack of mathematical knowledge, teacher education faculties should consider increasing a number of classes in general Mathematics and teaching methodology courses. Another solution might be in introducing some elective mathematics appreciation courses. We believe that the results of this research can be used as support to encourage some further investigations of the mathematics anxiety in pre-service primary teachers, but also investigating mathematics anxiety of students of different faculties, and finding ways of decreasing anxiety towards mathematics.

CONFLICT OF INTEREST

We authors declared that no competing interests exist.

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PROMOTING REFLECTION DURING MENTAL MATH

UDC: 37.011.3-052:51-8

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Abstract. Textual tasks, magic squares, mathematical riddles, puzzles and rebuses can be used as a tool for motivating and encouraging students' interest in mathematics. The aim of this paper is to suggest several such tasks that increase students' interest in mathematics and mathematical challenges, deepen their knowledge and encourage the process of acquiring long-lasting and structural knowledge among students.

Simultaneously, by pointing and discussing solutions we can increase students' mathematical thinking, promoting math reflection and encourage them to think about formulating new tasks that will have similar or different solutions.

1. INTRODUCTION

Physical activity is essential for the healthy and proper development of our body, in that direction the statement "healthy body healthy mind" is generally accepted. However, what do we know about the food for our mind? What do we need to do to train the mind and encourage logical thinking? What is mathematical thinking? How do we help the process of its formation?

Mathematical tasks and solving them are the "healthy food" for the mind. In this presentation, we will pay special attention to mathematical puzzles, logical tasks and games, textual tasks, tasks with strategies, symmetries, and tasks with tricks, i.e., different tasks that develop thinking and maintain our mind in perfect condition. It is important to note that the process of development and formation of mathematical concepts, development of thinking abilities, rational approach to problem solving, implementation of different ways of solving problems are development processes that can be assisted and guided. With appropriate, interesting tasks, we will try to "stimulate the process of thinking" and to manage the procedures necessary for solving the assigned tasks. In order to achieve maximum knowledge of every gifted and non-gifted student it is necessary to intensively form some kind of an ability during the sensitive period (period of highest receptiveness and development). The thinking process in a person begins when an intellectual or practical task emerges. Therefore, the qualities of thinking develop precisely while solving such tasks, and in school, they are posed to students not in a chaotic manner, but in a determined system of teaching tasks.

Teaching tasks develop students' thinking when well motivated, well understood, appropriate to the achieved level of intellectually, related to life problems. As stated above, thinking is a process of complex information processing, the end-result is "concepts-words" and "thinking-sentences." The teaching process involves empirical and theoretical thinking because it is a complete cognitive process through which students acquire the social and historical experience of humanity. Understanding that students adopt this experience in a short, generalised, structured and systemised form during the process of learning, is a problem for the corresponding part of empirical and theoretical thinking of learning through which this basic task of schools can be achieved.

All of this is in the function of improving the understanding and application of mathematical knowledge and skills, creating lasting knowledge and encouraging curiosity and love of mathematics. Given examples of tasks will encourage students not only to think, but also to want new mathematical challenges and new tasks.

The thesis of the interaction of the different types of thinking in the process of solving the teaching tasks does not contradict, but supports the character of the school activity as a complex scientific activity. Solving each teaching task requires not only simple processing of sensory material, confirming the facts (empirical opinion), but also getting to the essence of the situation of the relationships between them through specific complex mental activities for theoretical thinking, such as: creating situations in order to discover the common relationships in the studied system; modelling these relationships into graphic signs and forms in order to be studied in the same form, building a series of tasks of a general manner of solving; controlling and evaluating the manners to solve the teaching task.

Thinking is a process of complex information processing, the end-result is "concepts-words" and "thinking-sentences." Understanding is of great importance when trying to solve a teaching task. It is an active, multifaceted cognitive activity aimed at stating and discovering abstract connections between the new in the task and the imminent from the various subsystems of knowledge. As a result of this process, a new configuration of subsystems is born, which recognises (aha-moment), the adoption of the new from the given task.

Development of thinking and in particular the development of mental qualities — width, depth, independence, logic, mobility, concreteness, criticism, speed, creativity, target orientation, generalisation, insight, etc., . , is one of the most important and consistent goals and objectives of the teaching.

School mathematics, due to its specificity, possesses great opportunities for scholar's intellectual development which can be fully accomplished through prior organisation of the educational process. From this point of view, the conclusion of Vygotsky - Leontev's school of psychology according to which the child's development occurs in a process of adopting historically created mathematical knowledge, skills and habits is extremely important.

Mathematical tasks, the moment they enter the classroom, are intertwined with the educational aims, intentions and interactions between the teachers and the students. Therefore, tasks should not be considered as problems written in math text books or in the teachers' preparation, but should be considered as a classroom activity as well. Defined as activities, mathematical tasks in the educational process become connected and included in both training and teaching.

2. WHY DO WE NEED MENTAL MATH AND MENTAL MATH REFLECTION?

During mental math students

- Encourage mathematical thinking
- Stimulate curious children who want to explore
- Provide a mathematical record of problem situations and formation of mathematical models
- Foster love for mathematics and problem situations
- Encourage interest in challenges and assessments
- Stimulate creative thinking and students' motivation in they're learning
- Acquire long-lasting, structural knowledge among students

1. Continue the number sequence and determine the seventh term:
a) 1,2,3,4,….
b) 11,12,13,14, …
c) 1,1,2,2,3,3,4,4,5,5,
d) 2,4,6,8,10…
e) 1,3,5,7,9…

2. Continue the number sequences with one of the offered possible answers and explain your choice:
a) 1, 4, 10, 22, 46,…
Possible answers: 64, 86, 94, 122
b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$
Possible answers: $\frac{1}{14}, \frac{1}{4}, \frac{1}{12}, \frac{1}{8}$

3. Solve the rebus if same letters denote same digits:

$$\begin{array}{r} ЦАР \\ +\,ЦАР \\ \hline КРАЛ \end{array}$$

How many solutions are possible? Explain!

4. Solve the rebus if same letters denote same digits:

$$\begin{array}{r} A \\ BB \\ +\ A \\ \hline CCC \end{array}$$

5. There are two glasses with the same amount of white and red wine. We pour one-tenth of the red into the white wine, and then one-tenth of the resulting mixture back into the red wine.

Is there more white wine into the red wine or vice versa? Explain!

6. Three containers with no markings have volumes of 8, 5 and 3 l and the largest container is full. How do you measure an amount of 4 litres using the given containers?

3. EXAMPLES OF TASKS AS AN INTRODUCTION TO TASKS REDUCED TO IMPLEMENTATION OF LINEAR EQUATION

1. Which number is 4 times greater than 16?
2. 5 balls cost 85 denars. How much does one ball cost?
3. Nela has 564 denars. She has 236 denars more than her mother. She wants to buy a bag that costs 1000 denars. How much more money does she need?
4. 246 letters and 85 more postcards than letters were received in the post office. 110 fewer packages than letters were received. How many total items (letters, postcards and packages) were received in the post office?
5. Calculate the sum of the number 5 and its predecessor.
6. Find the difference between the number 9 and the predecessor of 3.
7. The sum of two numbers is 94. The larger number is 5 less than twice the smaller number. Find the two numbers.
8. Jana is 7 years older than her brother. In 5 years, the sum f their ages is 63. How old is each of them?
9. Petar had 6500 denars in banknotes of 50, 100 and 500 denars. He had an equal number of banknotes of every kind. How many banknotes of each kind did he has?
10. In a supermarket, the prices of food products decreased by 40%. If the price of a product is 180 denars, determine its price before the discount.
11. To build a fence around a rectangular garden requires 130 meters of wire. The length of the garden is 5 m larger than the width. Determine the dimensions of the garden.
12. A cyclist traveled 18 km during which time a pedestrian traveled 10 km If the cyclist was traveling at a speed of 9.6 km/h greater than the speed at which the pedestrian traveled, at what speed did each of them travel?
13. Two farmers can plow a field in 6 days. The first farmer can plow the field alone in 10 days. How many days will the second farmer need to plow the field alone?
14. The number 48 should be divided into two parts so that one part is three times larger than the other.
15. The numerator of a fraction is 2 less than the denominator. If the numerator decreases by 1 and the denominator increases by 1, the fraction ½ is obtained. Find the initial fraction.
16. The sum of three numbers is 54. Find the numbers if the first number is 4 larger than the doubled value of the second number and the third number is twice the first number.
17. In a mathematics test, the student must solve 20 tasks. The student receives 4 points for each task that is correct, and for each unsolved or

incorrectly solved task he loses 3 points. On that test, the student won a total of 38 points. How many tasks did the student solve correctly?

18. A worker can complete a job in 12 days. After working on that job for 3 days, another worker started to help who could complete the whole job in 15 days. How many days will they need to complete the job?

19. A man went on a journey by walking 30 km a day. 6 days later, another man followed the same route and after 9 days reached the first traveler. At what speed was the second traveler moving?

20. Sally is having a party. The first time the doorbell rings, 1 guest enters. The second time the doorbell rings, 3 guests enter. The third time the doorbell rings, 5 guests enter. Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

A. How many guests will enter on the 5 th ring? Explain or show how you found your answer

B. How many guests will enter on the 10th ring? Explain or show how you found your answer.

C. 19 guests entered on one of the rings. What ring was it?

New goals for students who solve logical tasks

(1) They learn to value mathematics
(2) They become confident in their ability to do mathematics
(3) They become mathematical problem solvers
(4) They learn to communicate mathematically
(5) They learn to reason mathematically

4. STAGES IN SOLVING LOGICAL TASKS

Stage 1: Understand the task (read the entire task or parts of it, drawing, sketch, symbolic representation of the task). This is the invisible stage and teachers usually skip it.

Stage 2: Build an idea and devise a plan to solve the task (this stage is connected with understanding the task).

Stage 3: Practical implementation of the devised plan (mathematical operations and solving the equation).

Stage 4: Examine the obtained solution (creative and interesting questions related to the task are asked additionally) such as: Is the obtained result correct? Why?

It is desirable to include as many students as possible in the classes, first using individual method of solving tasks and then making discussion. Namely, the previously stated and similar examples provide students with the opportunity to obtain not only wide-ranging operational knowledge and skills, but creating long-lasting structural knowledge. Increased level of this type of knowledge is, above all, conditioned by the fast-growing need for comprehensive and profound knowledge necessary to keep the active pace with the dynamic civilisation of the twenty first century.

CONCLUSIONS

Mathematical tasks and solving the same are an effective tool for developing mathematical activity and creativity among students. Reflection is associated with the "aha" moment, and it is necessary for creating long-lasting, structural knowledge among students. In this paper, appropriate tasks were selected which will further stimulate and motivate math reflection. I hope that this paper in which textual tasks, magic squares, mathematical riddles, puzzles and rebuses are used as a tool for motivating and encouraging students' interest in mathematics will motivate many teachers and contribute to maintaining mathematical talent among students and increasing love and interest in mathematics.

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ВЕКТОРИТЕ ВО ОСНОВНОТО ОБРАЗОВАНИЕ, ПРЕД И ПОСЛЕ ВОВЕДУВАЊЕТО НА КЕМБРИЏ ПРОГРАМАТА

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Ана Димовска, Томи Димовски

Апстракт. Во оваа статија е направена споредба на старата програма по математика и Кембриџ програмата од аспект на вектори. Ставен е акцент на потребата од изучување на векторите во основното образование. Мотивацијата за оваа статија, односно користењето на "векторскиот" пристап е развивање на геометриска интуиција за концептите како права, рамнина и простор; полесна интерпретација на проблеми од геометрија и навремено совладување на техника која е корисна од чисто математички цели (корисна во други математички области), но и во интердисциплинарни области како физика, хемија, компјутерски науки и други. Потребата од изучувањето на вектори во оваа статија е заокружена со елементарни примери од геометрија и физика.

1. Вовед

Каква е наставата по математика денес? Дали како наставници можеме да бидеме задоволни од програмите и нивната реализација? Наставата по математика е фокусирана на реализирање на наставната програма, така што главна цел на многу наставници е да ги натераат учениците да го научат пропишаниот материјал, во рамки на можностите. Со ваквиот начин на учење најзапоставени се најспособните ученици. Бидејќи не се доволно оптоварени, тие учат со леснотија и со леснотија го совладуваат материјалот. Како резултат на претходно кажаното, тие добиваат погрешен впечаток за тежината на материјата, а и не ретко губат интерес за истата. Предизвикот во посетување на настава и следење на истата се сведува на меморирање на предвидените по програма "методи" и задачи кои како такви и потекнуваат од концептот вектори, меѓутоа се адаптирани за една група на ученици која е само дел од целата група на ученици. Наставникот е оставен да ја организира наставата по Кембриџ која му е законска должност, но и по негова желба и зависно од неговите можности и ентузијазам да го потикнува нивниот развој. Овде лоцираме еден дел од проблемите во нашето образование.

Ако направиме една споредба на учебниците по математика за основно образование, по Кембриџ и старата програма, можеме да забележиме дека во некои теми тие целосно се разликуваат. Овде конкретно ќе ја разгледуваме геометријата бидејки по наше мислење тука има најголеми пропусти.

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Key words and phrases. Вектор, Кембриџ, Образование

За почеток ќе направиме мала дискусија за учебниците по програмата на Кембриџ. Нејзина главна карактеристика е спиралната наставна програма, односно учениците изучуваат одредена тема, но подоцна се навраќаат на истата тема и повторно ја изучуваат на повисоко ниво и во поинаков контекст. Идејата за постапно воведување на поимите и повторување на истите теми во второ полугодие, е сосема различно од претходната програма по која се водеше наставата по математика и дава можност на учениците кои не ја совладале или делумно ја совладале материјата, тоа да го корегираат. Од 6то до 9то одделение во глобала се изучуваат истите теми.

Во старата програма секоја година се воведуваше тема и таа се изучуваше до крај, без повторување и навраќање на претходните години. Избегнувањето на неколкукратното повторување отвара повеќе временски простор за воведување на разни рубрики како дополнителни и додадтни теми кои се соодветни за различни подгрупи во големата група од ученици. Конкретно, постоеа временски услови и можности да се работат теми кои се однесуваат на послабите, но и теми кои се однесуваат на посилните ученици.

2. Главен дел

Иако е иновативна Кембриџ програмата има некои пропусти, кои по наше мислење најмногу им прават штета на талентираните ученици, најмногу натпреварувачите. Темата вектори која воопшто не се изучува по програмата на Кембриџ, е еден од најголемите недостатоци во новата програма. Од друга страна, во старите учебници (слика 1,2 и 3) во 8мо одделение по деветтолетка или старо 7мо одделение по осмолетка, целосно се изучува темата вектори (поим вектор, операции со вектори, транслација и својства).

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Слика 1

ТЕМА 1. ВЕКТОРИ. ТРАНСЛАЦИЈА

ВЕКТОРИ. ОПЕРАЦИИ СО ВЕКТОРИ

ТРАНСЛАЦИЈА

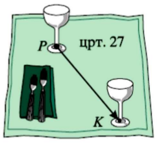
Слика 2

ТЕМА I
ВЕКТОРИ. ТРАНСЛАЦИЈА

Слика 3

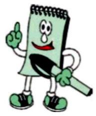
1. 2. 1. Транслација

Мира ја поместила чашата од едно на друго место на масата (црт. 27). Значи, Мира ја придвижила својата чаша од една во друга точка, односно извршила транслација. Придвижувањето на чашата е опишано со векторот $\vec{a}$, а тоа е всушност векторот кој е определен со двете точки: почеток P што одговара на почетната положба на чашата на Мира, и крај K што одговара на положбата во која чашата се наоѓа по придвижувањето. Вакво поместување, односно придвижувањето на даден објект се вика транслација.



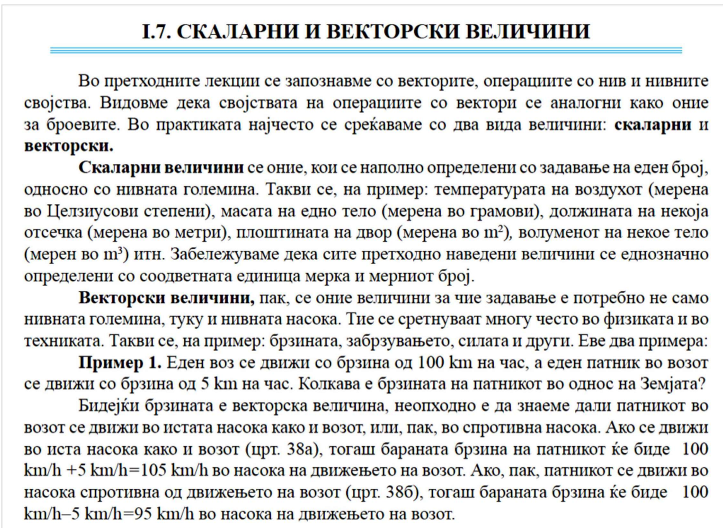
Примери за транслација во секојдневниот живот има многу. Токму од практичните потреби, транслацијата е дефинирана и во математиката, за понатаму да најде примена во многу други науки (најчесто во физиката и нејзините гранки). Транслацијата ќе ја дефинираме на следниот начин:

Нека $\vec{a}$ е даден вектор. Пресликувањето $f:\Pi \to \Pi$ кое на секоја точка M од рамнината Π придружува точка M_1 така што $\overrightarrow{MM_1} = \vec{a}$, односно $\overrightarrow{Mf(M)} = \vec{a}$ се вика **транслација**.



Слика 4

Познавањето на векторите и нивното користење во решавањето на геометриски проблеми од разни математички натпревари како и корелацијата со други предмети е од големо значење за учениците од основното образование. Иако темата вектори се изучува во прва година средно образование, сметаме дека е подобро таа да се изучува уште во 8мо одделение, како што беше во старата програма. Кој учебник да го одбереме по старата програма, Димовски, Крстевска, Ристовска (слика 1), Стефановски, Целаковски (слика 2) или Тренчевски, Тренчевски (слика 3), концептот е ист. Во сите учебници темата започнува со воведување на дефиниција за насочена отсечка или вектор и се разгледуваат неговите основни карактеристики должина, правец и насока. Потоа се воведуваат поимите колинеарни вектори, истонасочени вектори, спротивно насочени, спротивни вектори и поимот за нулти вектор. Сите овие поими се користат во програмата по физика за основно образование. Потоа е дадена дефиниција за еднаквост на вектори и како се пренесува вектор на дадена точка (добивање на еднаков вектор со зададен таков што неговата почетна точка да биде друга произволна точка). Дефинирани се операциите собирање и одземање вектори, како и множење на вектор со скалар. Потоа дефинирани се скаларни и векторски величини, учебник Тренчевски, Тренчевски (слика 5) кои се од голема важност за корелација на предметот математика со предметот физика. Брзина, забрзување, сила, движење и слично, сите тие се векторски величини кои се изучуваат по предметот физика во основното образование, а се векторски величини.



I.7. СКАЛАРНИ И ВЕКТОРСКИ ВЕЛИЧИНИ

Во претходните лекции се запознавме со векторите, операциите со нив и нивните својства. Видовме дека својствата на операциите со вектори се аналогни како оние за броевите. Во практиката најчесто се среќаваме со два вида величини: **скаларни** и **векторски.**

Скаларни величини се оние, кои се наполно определени со задавање на еден број, односно со нивната големина. Такви се, на пример: температурата на воздухот (мерена во Целзиусови степени), масата на едно тело (мерена во грамови), должината на некоја отсечка (мерена во метри), плоштината на двор (мерена во m^2), волуменот на некое тело (мерен во m^3) итн. Забележуваме дека сите претходно наведени величини се еднозначно определени со соодветната единица мерка и мерниот број.

Векторски величини, пак, се оние величини за чие задавање е потребно не само нивната големина, туку и нивната насока. Тие се сретнуваат многу често во физиката и во техниката. Такви се, на пример: брзината, забрзувањето, силата и други. Еве два примера:

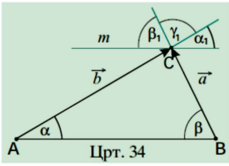
Пример 1. Еден воз се движи со брзина од 100 km на час, а еден патник во возот се движи со брзина од 5 km на час. Колкава е брзината на патникот во однос на Земјата?

Бидејќи брзината е векторска величина, неопходно е да знаеме дали патникот во возот се движи во истата насока како и возот, или, пак, во спротивна насока. Ако се движи во иста насока како и возот (црт. 38а), тогаш бараната брзина на патникот ќе биде 100 km/h +5 km/h=105 km/h во насока на движењето на возот. Ако, пак, патникот се движи во насока спротивна од движењето на возот (црт. 38б), тогаш бараната брзина ќе биде 100 km/h–5 km/h=95 km/h во насока на движењето на возот.

Слика 5

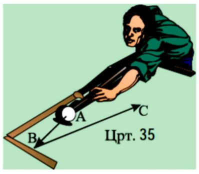
1. 2. 3. Примена на транслацијата

1. Докажи дека збирот на внатрешните агли во триаголникот изнесува 180^0.



Користи го црт. 34 и воочи ја врската при транслација на аголот α за вектор $\overrightarrow{AC} = \vec{b}$ и транслацијата на аголот β за вектор $\overrightarrow{BC} = \vec{a}$. Зошто $\alpha_1 + \beta_1 + \gamma_1 = 180^0$?

2. Запиши го векторот за кој ќе се помести топчето за билијард користејќи ја врската на векторите според точките означени на црт. 35.



3. Дадени се правите *p*, *q* и векторот $\vec{a}$ (црт. 36). Конструирај точки M и M_1 на правите *p* и *q*, соодветно, така што $M_1 = t_{\vec{a}}(M)$.

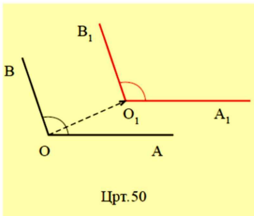
Слика 6

1.10. ПРИМЕНА НА ТРАНСЛАЦИЈАТА

Транслацијата наоѓа широка примена при докажувањето на некои тврдења и решавањето на некои конструктивни задачи.

***Теорема* 1. Ако краците на еден агол се истонасочени соодветно со краците на друг агол, тогаш тие два агла се едвакви.**

Претпоставка: $OA \| O_1A_1$ и $OB \| O_1B_1$ (црт. 50).
Тврдење: $\angle AOB = \angle A_1O_1B_1$.
Доказ. Да ја разгледаме транслацијата за вектор $\overrightarrow{OO_1}$, при која точката *O* се пресликува на точката O_1. При таа транслација полуправата *OA* ќе се преслика на истонасочената полуправа O_1A_1, а полуправата *OB* - на истонасочената полуправа O_1B_1, (црт. 50). Според тоа, аголот *AOB* при транслацијата за вектор $\overrightarrow{OO_1}$ ќе се преслика на аголот $A_1O_1B_1$, а тоа значи дека $\angle AOB = \angle A_1O_1B_1$.



Слика 7

На крајот од темата се изучува транслација, учебник Димовски, Крстеска, Ристовска (слика 4) и нејзините својства, како и примена на транслација. Лекцијата **својства на транслација** започнува со својството "При секоја транслација права се присликува во права паралелна со неа" и "При секоја транслација фигура се пресликува во фигура складна со неа", својства кои имаат широка примена во математиката. Со овие две својства многу полесно се совладуваат конструкциите, се докажува складност на фигури, итн. Многу од геометриските проблеми и познати резултати може многу поинтуитивно и поедноставно да се изучуваат и докажуваат со помош на вектори. Конкретно, многу интересни и важни тврдења во геометријата и конструктивни задачи се сведуваат на транслација. Во учебникот Димовски, Крстеска, Ристовска (слика 6) во лекцијата **примена на транслација** е докажано едно многу важно својство "Докажи дека збирот на внатрешните агли во триаголник е 180^0". Во учебникот Тренчевски, Тренчевски (слика 7) е докажана теоремата “Ако краците на еден агол се исто насочени соодветно со краците на друг агол, тогаш тие два агли се еднакви”. По усвојување на материјалот од оваа тема учениците се способни да го совладаат материјалот од предметот физика без да учат на памет, полесно да ги разберат концептите на сличност и складност на триаголници со кои ќе се сретнат во погорните оделенија, полесно можат да докажуваат својства и теореми, лесно можат да ја совладаат темата геометриски трансвормации и конструкциите на фигурите, како и да ја развијат нивната математичка интуиција при решавање на математички пробеми. Што се однесува на талентираните ученици и нивното учеството на натпревари од најниско до највисоко ниво, тоа е незамисливо и невозможно без да ја совладаат оваа тема. Учениците кои не учеле вектори во регуларното основно образование, мораат дополнително да трошат време за да ги совладаат, секако ако планираат да имаат некакви резултати.

Од друга страна, ако ги разгледаме содржините на учебниците по математика од 6то до 9то одделениепо Кембриџ програмата (слика 8-11), ќе забележиме дека секоја година се работат геометриски трансформации, односно осна симетрија, транслација, ротација, зголемување и сличност, но во ниедно одделение не се изучуваат векторите, барем не експлицитно. Исто така можеме да забележиме дека во 8мо одделение се изучуваат конструкции без претходно воведен поим вектор и без изучување на транслацијата векторски.

Содржина

Слика 8

Слика 9

Слика 10

Слика 11

По програмата на Кембриџ се изучува поместување на 2-де форми во насоките: горе, долу, лево и десно по однапред зададена мрежа, како и нивна ротација за 90^0, 180^0, 270^0 во насока на стрелките на часовникот и обратно. Се работат геометрсики трансформации, но без вектори. Во учебникот за 7мо одделение, во лекцијата **комбинирање геометриски трансформации** се напоменува својството "При геометриски трансформации добиената слика е секогаш складна со оригиналот" кое се докажува со помош на вектори (слика 12). Во лекцијата **транслација** на сликата која е дадена како пример за транслација користат ознаки за вектор (слика 13). Во истиот учебник ги има лекциите **цртање паралелни прави** и **цртање правоаголник и квадрат**, кои се всушност примена на транслација.

8.9 Комбинирање геометриски трансформации

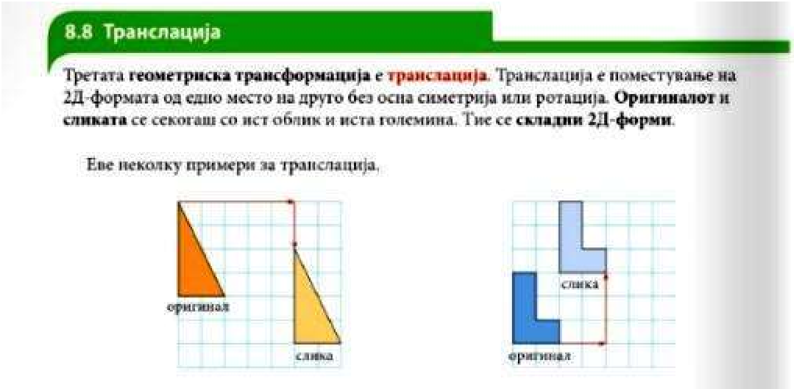
Геометриска трансформација е пресликување со кои од една форма (оригинал) добиваме друга форма (слика).

Геометриските трансформации применети на една форма ја менуваат нејзината местоположба.

Досега изучивме три геометриски трансформации: осна симетрија, ротација и транслација.

При овие геометриски трансформации, добиената слика е секогаш складна со оригиналот.

Слика 12



8.8 Транслација

Третата **геометриска трансформација** е **транслација**. Транслација е поместување на 2Д-формата од едно место на друго без осна симетрија или ротација. **Оригиналот** и **сликата** се секогаш со ист облик и иста големина. Тие се **складни 2Д-форми**.

Еве неколку примери за транслација.

Слика 13

Во учебникот за 8мо одделение во лекцијата **складни 2Д-форми**, на сликата која е дадена како пример за складни фигури повторно се користи ознаката за вектор, каде што точно се гледа дека при транслација на дадена фигура за даден вектор се добива складна фигура на дадената. Неколку лекции понатаму се изучува **конструкција на триаголник**, материја во која се користи транслацијата со вектор, но не и по Кембриџ програмата. Во лекцијата **триаголник и четириаголник** се спомнува својството за збир на аглите во триаголникот, кое видовме дека се докажува со помош на вектори. Во истиот учебник во лекцијата **геометриски трансформации** е решен пример на транслација. При решавањето иако не е објаснето со примена на вектори на сликата повторно се користи ознака за вектор.

3.3 Складни 2Д-форми

Во VI и VII одделение учевме за три геометриски трансформации на 2Д-формите. Тие се **транслација, осна симетрија** и **ротација.**

Почетната 2Д-форма пред трансформацијата ја нарекуваме **оригинал**.

Крајната 2Д-форма добиена по трансформацијата ја нарекуваме **слика**.

На цртежот е прикажана транслација за 3 квадратчиња надесно и 2 квадратчиња нагоре.

Кај оригиналот, страните што го образуваат правиот агол имаат должини од 3 квадратчиња и 2 квадратчиња, соодветно.

Кај сликата, соодветните страни имаат, исто така, должини 3 квадратчиња и 2 квадратчиња.

Кај оригиналот и кај сликата долниот лев агол е прав.

Хипотенузата на оригиналот е еднаква со хипотенузата на сликата.

Забележуваме дека сите соодветни страни и соодветни агли на двата триаголника се еднакви меѓу себе.

За ваквите 2Д-форми велиме дека се **складни**.

Слика 14

3.12 Геометриски трансформации

Во VI и VII одделение учевме за три вида **трансформации**, односно пресликувања.

пресликување во однос на права на симетрија

оригиналот е ротиран за даден агол, во дадена насока, околу центар на ротација (*A*).

оригиналот е поместен без ротација и без осна симетрија

Почетната 2Д-форма ја нарекуваме **оригинал**.

Крајната 2Д-форма ја нарекуваме **слика**.

Во секоја од овие три трансформации сликата е **складна** на оригиналот. Тоа значи дека сликата има иста големина и форма со оригиналот.

Слика 15

Во учебникот за 9то одделение во лекцијата **опишување трансформации** конечно се спомнува поимот вектор (слика 16): „Транслација за вектор (x,y) значи поместување за x единици надесно или налево (по x-оската) и за y единици нагоре или надолу (по y-оската)". Тука го дефинира поимот транслација преку поимот вектор, иако претходно не е дефиниран поимот вектор.

Транслација

Треба да е даден детален опис за поместувањето.

Тоа често се претставува како растојание надесно (или налево) и растојание нагоре (или надолу).

За насоките можеме да користиме и запис со вектор, како на пример: (x, y)

Транслација за вектор (x, y) значи поместување за x единици надесно или налево и за y единици нагоре или надолу.

Ако вредноста на x е позитивна, тогаш поместувањето е надесно.

Ако вредноста на x е негативна, тогаш поместувањето е налево.

Ако вредноста на y е позитивна, тогаш поместувањето е нагоре.

Ако вредноста на y е негативна, тогаш поместувањето е надолу.

На пример, поместување за вектор $(-4, -5)$ значи поместување за 4 единици налево и за 5 единици надолу.

Слика 16

Од досегашното излагање можеме да заклучиме дека во новата програма поимот вектор, воопшто не е дефиниран, нити се спомнува дека транслацијата е всушност поместување за даден вектор. Со избегнувањето на векторскиот пристап, на некој начин се ограничуваат можностите и размислувањето на учениците. Се избегнува користење на поим кој во иднина секако ќе мора да го сретнат. Поим кој им е потребен за усовршување на математичките способности и совладување на темите по предметот физика.

На цртежи, силите ги претставуваме со насочена отсечка или вектор.

Користењето **вектор на сила** е соодветен начин за претставување на силите бидејќи дава информации за правецот и насоката на силата, како и за нејзината големина.

Цртање на векторот на силата

Со векторот на силата се прикажува правецот, насоката и големината на силата. Со означувањето на векторот се објаснува кое тело дејствува со сила и врз кое тело таа сила дејствува.

На сликата е даден пример во кој девојка турка количка. Со означувањето на векторот на силата се покажува кое тело дејствува со сила (турка) и врз кое тело таа сила дејствува.

Сили се појавуваат кога две тела заемнодејствуваат, односно взаемно дејствуваат.

Магнетот може да привлече железен клинец. Велиме дека магнетот и клинецот си заемнодејствуваат. Магнетот го влече клинецот, односно клинецот е привлечен од магнетот.

На сликата е прикажана силата со која магнетот дејствува НА клинецот.

Слика 17

Активност 1.1 Означување на силите

Пронајди неколку сили и означи ги со вектори.

1 Исечи три стрелки од картон или хартија. Нека бидат долги околу 20 cm. Употреби ги за да ги прикажеш векторите на силите кај некои заемнодејства.
2 Најди место или предмети на кои дејствува сила. Одреди го правецот и насоката на дејство на силата.
3 Запиши соодветна ознака на еден од векторите (кое тело дејствува со сила и на кое тело таа сила дејствува).
4 Постави ја или залепи ја стрелката така што ќе ги означува насоката, правецот на дејство и нападната точка на силата.
5 Повтори ја постапката и со другите стрелки.

слика 18

Кембриџ програмата не е усогласена по предметите математика и физика. Во физиката по Кембрич програмата, како и по старата програма се изучуваат векторски величини и се очекува дека учениците знаат што е поимот вектор. Уште во првата тема се изучува силата како векторска величина (слика 17). На учениците им е зададена активност (слика 18) која треба да ја извршат со помош на вектори, а тие воопшто се немаат сретнато со тој поим. Од нив се очекува самите дополнително да совладаат цела тема, за да можат да ја пратат програмата по физика.

4. Заклучок

Заклучокот што треба да го извлечеме по оваа направена споредба меѓу старата и новата програма е дека иако Кембриџ програмата има свои предности таа има голем недостаток во областа геометрија. Очигледно е дека Кембриџ програмата не придонесува за некаков поголем интелектуален развој, ако се знае дека голем дел од содржините децата ги усвојуваат механички и фактографски, вклучувајќи го помнењето, наместо да ги развиваат мисловните процеси. Конкретно темата вектори има голема важност во развивањето на математичките способности и размислувањето на секој ученик. Затоа сметаме дека наставниците не може да се држат само до пропишаниот материјал, туку мора да изнаоѓаат начин и време за да воведат и други поими кои не се предвидени во програмата, се со цел да подобри моменталниот образовен процес.

Судир на интереси

Авторите изјавија дека нема судир на интереси.

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NOVEL APPROACH TO STUDENTS FOR EFFECTIVENESS AND EFFICIENCY IN MATH EDUCATION

UDC: 37.011.3-051:51]:159.944.075

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Abstract. There are numerous proven effective methods for corresponding math knowledge transfer as well as arguments to which methods correspond to certain circumstances within math education. This work's focus, however, is not upon WHAT is used as methods, neither upon WHY those methods are used, but rather upon HOW the teacher may effectively and efficiently use whichever methods he or she finds appropriate. The novel approach is suggested, which is expected to enable significant improvement in the outcome of the math knowledge transfer, without any limitation to the set of methods that are used by the teacher. The approach is individually created with regard to the notion of Drivers and Working styles and Transactional Analysis as recognized and widely used personality theory. The communication doors of the students relevant to behaviour, feelings and thinking are discussed as well as sentences patterns as indicators to dominant student Drivers and/or Working styles.

The results and conclusions are summarized according to several students' Working styles categories.

1. INTRODUCTION TO THE CONCEPT OF DRIVERS AND WORKING STYLES

The theory of Drivers has been introduced about half a century ago by Taibi Kahler [14], and has been developed into five characteristic styles since. Kahler named the Drivers after Freud's drive, or basic instinct for repetitive behaviour. He defined them as programmed responses to the messages we carry in our heads, that we have subconsciously adopted from important people in our past (parents or other parental figures, including teachers) as in Fig.1, manifested as a certain set of a person's compulsive behaviours, particularly when the person is under stress [11],[14]. In fact, Drivers are unconscious behavioural patterns, which affect each segment of our lives, regardless to whether we are at home, at school, at work, alone, or with anyone. They are subconscious attempts by us to behave in ways that will gain us the recognition we need from others [3],[15]-[17],[24]. Drivers represent a type of survival mechanisms or subconscious mental strategies that we develop to counterbalance injunctions.

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Key words and phrases. psychology of math education, Working styles

This concept had later been developed and elaborated by many researchers: Gellert, Silver, Tudor and others [19]. Drivers' characteristics are very specific and can be both positive and negative, their orientation can be from or toward people [3]-[6],[22],[24]. They have specific behavioral indicators (words, voice, posture, facial expressions and gestures). Drivers may be observed as preferred styles of social interaction in contact, and as specific reactions to problems and stress. An early review to the positive aspects of Drivers has been given by Klein, however Hay is to one to elaborate and focus specifically on the positive aspects and has given the name Working styles to such aspect of the Drivers [12],[13]. Hay has created her well known Questionnaire to identify the concept of person's Working styles under professional conditions.

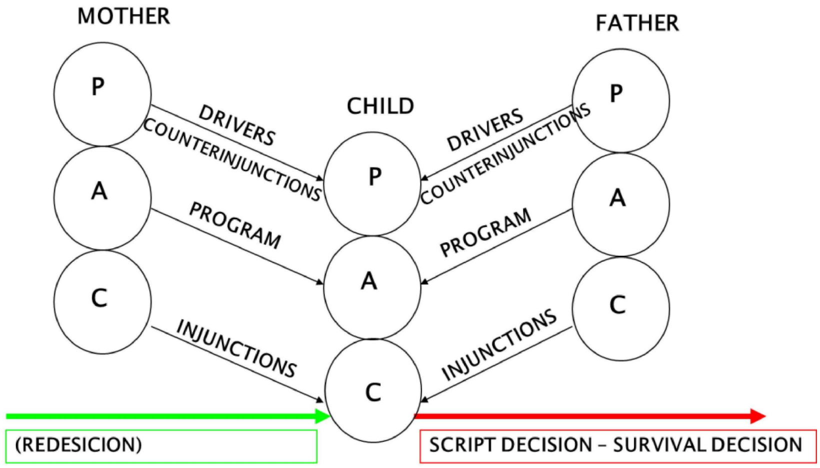


Figure 1. Scheme for developing dominant Drivers (Working styles)

Identifying the Drivers (Working styles) that an individual is manifesting enables the possibility for the individual to recognize and develop the potential of the positive aspects of their behavior and to constructively react to the negative ones. This work is focused on the Working styles only as the positive aspects of Drivers, and the theory is used to offer a novel approach to math students for improving and sustaining their work effectiveness and efficiency.

There are five identified Working styles with their characteristics, benefits and drawbacks, and they are named after the characteristic behavior manifested [3]-[6],[13],[22],[24],[25]:

- Be Perfect
- Be Strong
- Try Hard

- Hurry Up
- Please Others

In a person's real life, there is usually an influence of two Working styles [22],[24]. Such a combination seems to be in accordance with the experience, and people tend to a combination of two (sometimes and more rare three) Working styles. Some research and publications show that each profession, based on previous statistical evaluation, has a highly predictive presence of specific dominant Working style (Driver) [19]. For example, for the mathematicians, Be Perfect was identified as the primary dominant Driver, which is "justified" by necessity of having strong logic, organizing skills and ability to recognize and synthesize facts (Kahler, 2006), whereas Try Hard is present as a secondary dominant Driver. In contrast to that, the Try Hard Driver is not present at all as a dominant Driver in the profession Legal Advisor. It could be explained by the clearly established principles within the profession in the form of laws and regulations which does not require finding new and innovative solutions. Within the professional selection of staff, the concept of Working styles can be used as a tool for verifying the presence of the necessary skills and abilities in accordance with the job qualification. Within the professional selection of staff, the concept of Drivers can be used as a tool for verifying the presence of the necessary skills and abilities in accordance with the job qualification. Kahler in 2013 has developed and used PCM (in which the basis is the drivers) in the selection of astronauts for NASA (National Aeronautics and Space Administration) for more than 10 years. The concept of Working styles is also applicable in the field of employee motivation [2]-[6]. Kahler summarizes the incentive strategies that can be used to direct people towards optimal performance.

2. THEORY, METHODS AND DISCUSSIONS

The main characteristics of the Working styles will be discussed with regard to their behavioural features [22],[24], their communication doors opening order, and correspondingly summarized in tables. Positive values for each Working Style are given in each description.

2.1. WORKING STYLE BE PERFECT (BP)

If one may answer positively to the following questions: *Do you pride yourself on your accuracy? Does it worry you when you see mistakes? Do you enjoy the challenge of bringing order into the world?*, then he/she may find to be driven by the Be Perfect Working Style. Specific characteristics are presented in Table 1.

Table 1. BP characteristics

WS	Words	Tones	Gestures	Postures	Facial Expressions
BE PERFECT	"of course" "obviously" "efficacious" "clearly" "I think" (tells more than asked) "	clipped, righteous	counting on fingers, cocked wrist, scratching head	erect, rigid	stern, shame, embarrassment

The Communication doors that this person communicates through, open by the following order:

1. Thinking (T) communication door
2. Feelings (F) communication door
3. Behavior (B) communication door

This would help us to communicate easily to a BP person if the communication doors order would be followed. Positive values of this WS may be making a real hit and being precise, while permissions and advices in everyday work to be given are to make a point, to define time limits for completing the work or to define content and aims.

2.2. Working style Hurry Up (HU)

If one may answer positively to the following questions: *Do you enjoy having lots to do? Are you usually in a hurry? Can you pull out all the stops when urgent work comes up?*, then he/she may find to be driven by the Hurry Up Working Style. Specific characteristics are presented in Table 2.

Table 2. HU characteristics

WS	Words	Tones	Gestures	Postures	Facial Expressions
HURRU UP	"let's go" interrupts people-finishes their sentences	up & down	squirms, taps fingers	moves quickly	frowning, eyes shifting, rapid

There is no specific order in Communication doors. The main reason for this is that HU may not be a dominant Driver, and the contact door is determined by the dominant Driver.

Positive values of this WS may be being aware of time, while permissions and advices in everyday work to be given are to take as much time as he/she needs, to make a break and take a rest between two different works,or to plan and define their priorities.

2.3. WORKING STYLE BE STRONG (BS)

If the person finds himself/herself familiar to the following statements: *You pride yourself on your ability to cope. You may even welcome pressure because it gives you the chance to show how well you can deal with it. You stay calm when there is a crisis*, then he/she may find to be driven by the Be Strong Working Style. Specific characteristics are presented in Table 3.

Table 3. BS characteristics

WS	Words	Tones	Gestures	Postures	Facial Expressions
BE STRONG	"no comment" "I don't care" doesn't use here-and-now feelings	hard, monotone	hands rigid, arms folded	rigid, one leg over	plastic, hard, cold

The Communication doors that this person communicates through, open by the following order:

1. Behavior (B) communication door
2. Thinking (T) communication door
3. Feelings (F) communication door

With regard to successful communication, the BS person may be reached mostly by questions/statements considering the behavior, and such person will not be easily responsive when asked about feelings.

Positive values of this WS are reacting correspondingly in problematic situations, while permissions and advices in everyday work to be given are to experience and verbalize their feelings, to listen to their feelings and to express them in a corresponding way.

2.4. WORKING STYLE PLEASE OTHERS (PO)

If the person can give a positive answer to the following questions: *Is your priority to get on well with people? Are you intuitive about how people are feeling? Are you happiest working in a team where everyone's views are taken into account?*, then he/she may find to be driven by the Please Others Working Style. Specific characteristics are presented in Table 4.

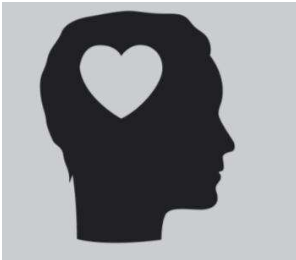


Table 4. PO characteristics

WS	Words	Tones	Gestures	Postures	Facial Expressions
PLEASE OTHERS	"You know" "Could you" "Can you" "Kinda" "Um Hmm" "Would you"	high whine	hands outstretched, head nodding frequently	head nodding	raised eyebrows, looks away

The Communication doors that this person communicates through, open by the following order:

1. Feelings (F) communication door
2. Behavior (B) communication door
3. Thinking (T) communication door

If we tend to successful communication, the BS person may be reached mostly by questions/statements considering the feelings, and such person will not be easily responsive when asked about their thinking.

Positive values of this WS are being highly emphatic to other people, while permissions and advices in everyday work to be given are to take care of their own feelings, to take some time to think and to think about what they want for a mutual success.

2.5. WORKING STYLE TRY HARD (TH)

If the person can give a positive answer to the following questions: *Are you motivated by almost anything as long as it's new? Do you enjoy most the early stages of each new project or task? Is it a challenge to explore different areas of work?*, then he/she may find to be driven by the Try Hard Working Style. Specific characteristics are presented in Table 5.

Table 5. TH characteristics

WS	Words	Tones	Gestures	Postures	Facial Expressions
TRYHARD	"It's hard" "I can't" "I'll try" "I don't know" (doesn't answer questions-repeats, tangents)	impatient	clenched, moving fists	sitting forward, elbows on legs	slight frown, perplexed look

The Communication doors that this person communicates through, open by the following order:

1. Behavior (B) communication door
2. Feelings (F) communication door
3. Thinking (T) communication door

If we tend to successful communication, the TH person may be reached mostly by questions/statements considering the behavior, and such person will not be easily responsive when asked about their thinking.

Positive values of this WS are being active and doing their work, while permissions and advices in everyday work to be given are to tell them when it is enough, to make a break, to organize their time to get some rest, to set the work within time intervals with high level of energy, to create an appropriate (peaceful) working atmosphere, and that it is necessary to evaluate the time needed.

3. DRIVERS AND SCRIPT PATTERNS

Considering the fact that each individual is driven by mainly two Working styles, the combination of the two implies some specific characteristics which influence the way of living lives and thinking, feeling, and behaving in a certain pattern that is named a life script pattern. Taibi Kahler describes 6 script patterns (of which 5 that are most common will be subject of our interest) that influence the individual thinking, feeling and behaving manner. This theory perfectly applies for students who are part of math education. Considering the time aspect, these script patterns are related to our perception of time and manner in which we tend to focus to our past, to our present or our future [7].

According to d-r Kahler, the script processes connected to these patterns are:

AFTER – "I am afraid something bad will happen.";

UNTIL – "I can't have fun until...";

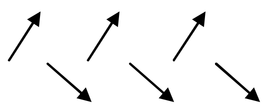
ALWAYS – "Feeling of being cornered", blames or waits for a rescue, or manipulates others from a position of "being cornered";

NEVER – Has difficulties of finishing life projections;

ALMOST – Almost completes the work, but not all of it.

Each of the Working styles may be found within the corresponding script pattern that has specific characteristics [1],[2],[22]. Most of us follow one script pattern in each aspect of life. However, there are people who follow one script pattern in their private life, and some other in their professional or social life. It is important to point out that the script pattern is being expressed or manifested in the person's lifestyle, as well as in their overall life plan. A lot may be read in literature [7]-[10],[12],[13],[18],[20]-[24] and further the main characteristics considering particularly the sentence patterns of the most common life scripts will be summarized as presented in Table 6.

Table 6. Characteristic sentence pattern for different life scripts

Script patterns (combination of WS)	**Characteristic sentence pattern**
NEVER (TH, rarely others)	 - Discontinued, seem like it will never end
ALWAYS (BS, HU, sometimes others)	- Nonconsistent sentences - A lot of qualifying words (maybe, we'll see, I'm not sure, sometimes …)