Junior Balkan MO 2005

Veria, Greece

$\boxed{1}$ Find all positive integers x, y satisfying the equation

$$9(x^2 + y^2 + 1) + 2(3xy + 2) = 2005.$$

$\boxed{2}$ Let ABC be an acute-angled triangle inscribed in a circle k. It is given that the tangent from A to the circle meets the line BC at point P. Let M be the midpoint of the line segment AP and R be the second intersection point of the circle k with the line BM. The line PR meets again the circle k at point S different from R.

Prove that the lines AP and CS are parallel.

$\boxed{3}$ Prove that there exist

(a) 5 points in the plane so that among all the triangles with vertices among these points there are 8 right-angled ones;

(b) 64 points in the plane so that among all the triangles with vertices among these points there are at least 2005 right-angled ones.

$\boxed{4}$ Find all 3-digit positive integers $\overline{abc}$ such that

$$\overline{abc} = abc(a + b + c),$$

where $\overline{abc}$ is the decimal representation of the number.