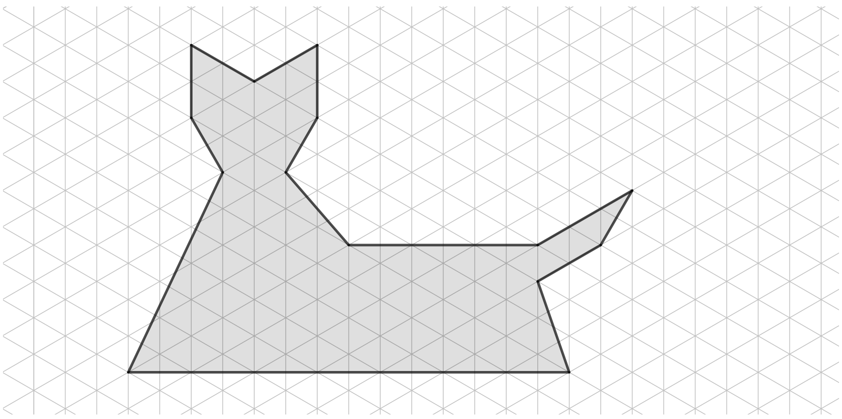
7th Iranian Geometry Olympiad

Elementary level

October 30, 2020

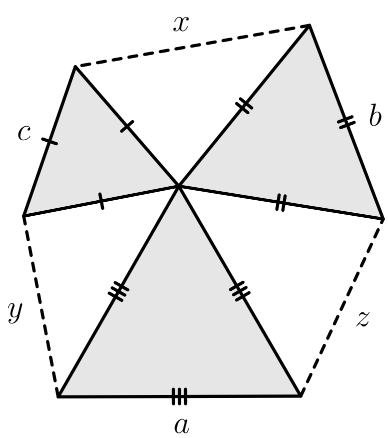
The problems of this contest are to be kept confidential until they are posted on the official IGO website: igo-official.ir

Problem 1. By a *fold* of a polygon-shaped paper, we mean drawing a segment on the paper and folding the paper along that. Suppose that a paper with the following figure is given. We cut the paper along the boundary of the shaded region to get a polygon-shaped paper.
Start with this shaded polygon and make a rectangle-shaped paper from it with at most 5 number of folds. Describe your solution by introducing the folding lines and drawing the shape after each fold on your solution sheet.
(Note that the folding lines do not have to coincide with the grid lines of the shape.)



Problem 2. A parallelogram $ABCD$ is given. Points E and G are chosen on CD such that AC is the angle bisector of both angles $\angle EAD$ and $\angle BAG$. The line BC intersects AE and AG at F and H, respectively. Prove that the line FG passes through the midpoint of HE.

Problem 3. According to the figure, three equilateral triangles with side lengths a, b, c have one common vertex and do not have any other common point. The lengths x, y and z are defined as in the figure. Prove that $3(x + y + z) > 2(a + b + c)$.



Problem 4. Let P be an arbitrary point in the interior of triangle ABC. Lines BP and CP intersect AC and AB at E and F, respectively. Let K and L be the midpoints of the segments BF and CE, respectively. Let the lines through L and K parallel to CF and BE intersect BC at S and T, respectively; moreover, denote by M and N the reflection of S and T over the points L and K, respectively. Prove that as P moves in the interior of triangle ABC, line MN passes through a fixed point.

Problem 5. We say two vertices of a simple polygon are *visible* from each other if either they are adjacent, or the segment joining them is completely inside the polygon (except two endpoints that lie on the boundary). Find all positive integers n such that there exists a simple polygon with n vertices in which every vertex is visible from exactly 4 other vertices.
(A simple polygon is a polygon without hole that does not intersect itself.)

Time: 4 hours.
Each problem is worth 8 points.