Language: English

Problem 1. The real numbers a, b, c, d satisfy simultaneously the equations

$$abc - d = 1,\ bcd - a = 2,\ cda - b = 3,\ dab - c = -6.$$

Prove that $a + b + c + d \neq 0$.

Problem 2. Find all integers $n, n \geqslant 1$, such that $n \cdot 2^{n+1} + 1$ is a perfect square.

Problem 3. Let AL and BK be angle bisectors in the non-isosceles triangle ABC (L lies on the side BC, K lies on the side AC). The perpendicular bisector of BK intersects the line AL at point M. Point N lies on the line BK such that LN is parallel to MK. Prove that $LN = NA$.

Problem 4. A 9×7 rectangle is tiled with tiles of the two types shown in the picture below (the tiles are composed by three, respectively four unit squares and the L-shaped tiles can be rotated repeatedly with $90°$).

Let $n \geqslant 0$ be the number of the 2×2 tiles which can be used in such a tiling. Find all the values of n.

Each problem is worth 10 points.
Alloted time: $4\,{}^1\!/_2$ hours.