

XXIII MMC 2020

Problem 1

Let a, b, c be positive real numbers such that $a + b + c = 4$. Prove that

$$\frac{ab}{\sqrt[4]{3c^2 + 16}} + \frac{bc}{\sqrt[4]{3a^2 + 16}} + \frac{ca}{\sqrt[4]{3b^2 + 16}} \leq \frac{4}{3} \sqrt[4]{12}.$$

Problem 2

Let S be a set of $n \geq 2$ positive integers. Prove that there exist at least n^2 integers that can be written in the form $x + yz$ with $x, y, z \in S$.

Problem 3

Determine all integers $m \geq 2$ for which there exists an integer $n \geq 1$ with

$$\gcd(m, n) = 1 \quad \text{and} \quad \gcd(m, 4n + 1) = 1.$$

Problem 4

Let P, Q and R three points on a circle k_1 , such that $PQ = PR$ and $PQ > QR$. Let k_2 be the circle with centre P that passes through Q and R . Suppose that the circle with centre Q and passing through R intersect k_1 again at X and k_2 again at Y .

The points X and R lie on different sides of the line PQ .

Prove that P, X and Y lie on a line.

Секоја задача се вреднува по 7 поени.

Време за работа 4:30

Користењето на калкулатор не е дозволено.