Mediterranean mathematical olympiad
29.04.2018

Problem 1.

An integer $a \geq 1$ is called *Aegean*, if none of the numbers $a^{n+2} + 3a^n + 1$ with $n \geq 1$ is prime. Prove that there are at least 500 Aegean integers in the set $\{1, 2, \ldots, 2018\}$.

Problem 2.

Let $a_1, a_2, \ldots, a_n$ *be* $n \geq 2$ *real numbers such that* $0 \leq a_i \leq \pi/2$. *Prove that*

$$\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\sin a_i}\right)\left(1+\prod_{i=1}^{n}(\sin a_i)^{1/n}\right) \leq 1.$$

Problem 3.

Determine the largest integer N, for which there exists a $6 \times N$ table T that has the following properties:

(i) Every column contains the numbers $1, 2, \ldots, 6$ in some ordering.

(ii) For any two columns $i \neq j$, there exists a row r such that $T(r,i) = T(r,j)$.

(iii) For any two columns $i \neq j$, there exists a row s such that $T(s,i) \neq T(s,j)$.

Problem 4.

ABC is an acute triangle. AE and AF are isogonal cevians, where $E \in BC$ and $F \in BC$. The straight lines AE and AF intersect again the circumcircle of ABC at points M and N, respectively. In the rays AB and AC we get points P and R such that $\measuredangle PEA = \measuredangle B$ and $\measuredangle AER = \measuredangle C$. Let $L = AE \cap PR$ and $D = BC \cap LN$.

Prove, with reasons, that

$$\frac{1}{MN} + \frac{1}{EF} = \frac{1}{ED}.$$

Each problem is worth 7 points.
Time allowed 4:30
The use of calculator is not allowed