7$^{\text{th}}$ Iranian Geometry Olympiad

Advanced level

October 30, 2020

The problems of this contest are to be kept confidential until they are posted on the official IGO website: igo-official.ir

Problem 1. Let M, N, and P be the midpoints of sides BC, AC, and AB of triangle ABC, respectively. E and F are two points on the segment BC so that $\angle NEC = \frac{1}{2}\angle AMB$ and $\angle PFB = \frac{1}{2}\angle AMC$. Prove that $AE = AF$.

Problem 2. Let ABC be an acute-angled triangle with its incenter I. Suppose that N is the midpoint of the arc BAC of the circumcircle of triangle ABC, and P is a point such that $ABPC$ is a parallelogram. Let Q be the reflection of A over N, and R the projection of A on QI. Show that the line AI is tangent to the circumcircle of triangle PQR.

Problem 3. Assume three disjoint circles with the property that every line separating two of them have intersection with the interior of the third one. Prove that the sum of pairwise distances between their centers is at most $2\sqrt{2}$ times the sum of their radii.
(A line separates two circles, whenever the circles do not have intersection with the line and are on different sides of it.)

Note. Weaker results with $2\sqrt{2}$ replaced by some other c may be awarded points depending on the value of $c > 2\sqrt{2}$.

Problem 4. Convex circumscribed quadrilateral $ABCD$ is given such that its incircle is tangent to AD, DC, CB, and BA at K, L, M, and N. Lines AD and BC meet at E and lines AB and CD meet at F. Let KM intersects AB and CD at X and Y, respectively. Let LN intersects AD and BC at Z and T, respectively. Prove that the circumcircle of triangle XFY and the circle with diameter EI are tangent if and only if the circumcircle of triangle TEZ and the circle with diameter FI are tangent.

Problem 5. Consider an acute-angled triangle ABC ($AC > AB$) with its orthocenter H and circumcircle Γ. Points M and P are the midpoints of the segments BC and AH, respectively. The line AM meets Γ again at X and point N lies on the line BC so that NX is tangent to Γ. Points J and K lie on the circle with diameter MP such that $\angle AJP = \angle HNM$ (B and J lie on the same side of AH) and circle ω_1, passing through K, H, and J, and circle ω_2, passing through K, M, and N, are externally tangent to each other. Prove that the common external tangents of ω_1 and ω_2 meet on the line NH.

Time: 4 hours and 30 minutes.
Each problem is worth 8 points.