

Junior Balkan MO 2003

Kusadasi, Turkey

- [1] Let n be a positive integer. A number A consists of $2n$ digits, each of which is 4; and a number B consists of n digits, each of which is 8. Prove that $A + 2B + 4$ is a perfect square.
- [2] Suppose there are n points in a plane no three of which are collinear with the property that if we label these points as A_1, A_2, \dots, A_n in any way whatsoever, the broken line $A_1 A_2 \dots A_n$ does not intersect itself. Find the maximum value of n .

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- [3] Let D, E, F be the midpoints of the arcs BC, CA, AB on the circumcircle of a triangle ABC not containing the points A, B, C , respectively. Let the line DE meet BC and CA at G and H , and let M be the midpoint of the segment GH . Let the line FD meet BC and AB at K and J , and let N be the midpoint of the segment KJ .
- a) Find the angles of triangle DMN ;
- b) Prove that if P is the point of intersection of the lines AD and EF , then the circumcenter of triangle DMN lies on the circumcircle of triangle PMN .

- [4] Let $x, y, z > -1$. Prove that

$$\frac{1+x^2}{1+y+z^2} + \frac{1+y^2}{1+z+x^2} + \frac{1+z^2}{1+x+y^2} \geq 2.$$

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