

Student

3-Point-Problems

1. **S 3.3** For which of the following values of x is the value of the expression $\frac{x^2}{x^3}$ the smallest?

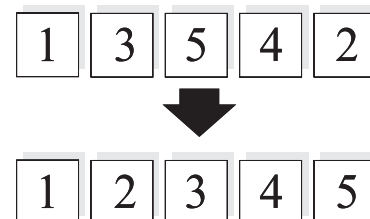
- (A) 2 (B) 1 (C) -1 (D) -2 (E) -3

2. **S 3.4** How many numbers from 2 to 100 are equal to the cube of an integer?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. **S 3.33** Five cards numbered from 1 to 5 are arranged as shown. Per move, any two cards may be interchanged. Find the smallest number of moves required to arrange them in increasing order.

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

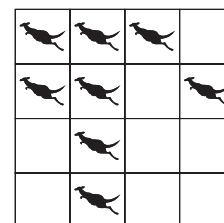


4. **S 3.5** If $888 \cdot 111 = 2 \cdot (2 \cdot n)^2$, and n is a positive integer, n equals

- (A) 8 (B) 11 (C) 22 (D) 111 (E) 444

5. **S 3.30** There are eight kangaroos in the squares of the table as shown. A kangaroo can be moved from its square directly into any empty square. Find the least number of moves so that exactly two kangaroos remain in any row and in any column of the table.

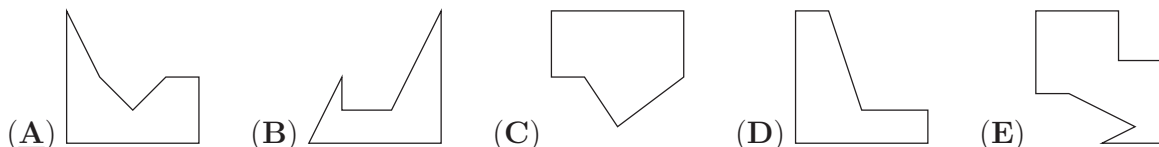
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



6. **S NP.1** A square piece of paper has been cut into 3 pieces. Two of them are



What is the shape of the third piece?

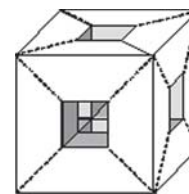


7. **S 3.22** The sum of four consecutive positive integers cannot be equal to

- (A) 2002 (B) 22 (C) 202 (D) 222 (E) 220

8. **S 3.31** A $3 \times 3 \times 3$ cube weighs 810 grams. If we drill three holes through it as shown, each of which is a $1 \times 1 \times 3$ rectangular parallelepiped, the weight of the remaining solid is

- (A) 540 g (B) 570 g (C) 600 g (D) 630 g (E) 660 g

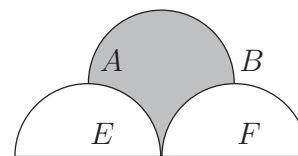


9. **S 3.51** If f is a function such that $f(x+1) = 2f(x) - 2002$ holds for all integer values of x and $f(2005) = 2008$, then $f(2004)$ equals:

- (A) 2004 (B) 2005 (C) 2008 (D) 2010 (E) 2016

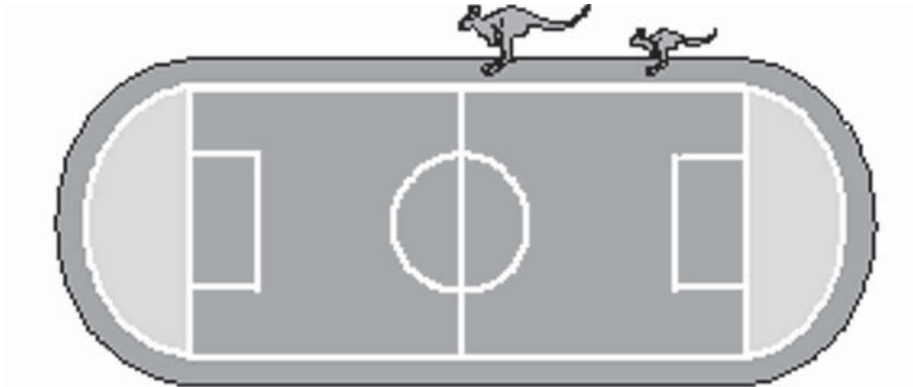
10. **S 3.48** We are given three semi-circles as shown. $ABEF$ is a rectangle and the radius of each of the semi-circles is 2 cm. E and F are the centers of the bottom semi-circles. The area of the shaded region in cm^2 is

- (A) 8 (B) 7 (C) 2π (D) $2\pi + 1$ (E) $2\pi + 2$



4-Point-Problems

11. S 3.27 Mom Kangaroo and her child Jumpy are jumping around a stadium with perimeter 330 m. They each make one leap per second. Mom leaps 5 m each time, and Jumpy 2 m. They both start at the same point and jump in the same direction.



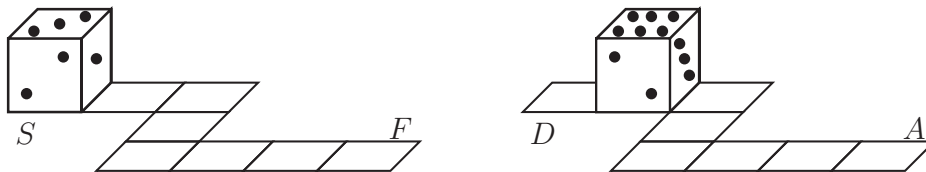
After 25 seconds Jumpy gets tired and stops while his mother continues. How long does it take her to overtake Jumpy?

- (A) 15 s (B) 24 s (C) 40 s (D) 51 s (E) 66 s

12. S 4.39 Henny paints each face of several wooden cubes white or black, using both colours on each cube. How many different colourings are possible?

- (A) 8 (B) 16 (C) 32 (D) 52 (E) 64

13. The sum the dots on opposite faces of a die always equals 7. A die rolls as shown below.



At the starting point (D) the top face is 3. Which will be the face at the end point (A)?

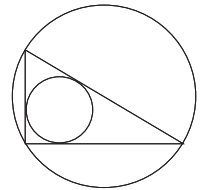
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

14. S 4.58 A box contains 60 tickets: some red, some blue and some white. If all red tickets were replaced by blue tickets, then there would be twice as many blue tickets as white tickets; but if all the white tickets were replaced with blue ones, then there would be three times as many blue tickets as red tickets. The number of blue tickets in the box is

- (A) 10 (B) 15 (C) 20 (D) 25 (E) 30

15. S 4.38 Let a and b be the lengths of the sides of the right-angled triangle. If d is the diameter of the incircle and D is the diameter of the circumcircle of this triangle, then $d + D$ is equal to

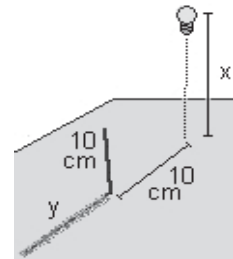
- (A) $a + b$ (B) $2(a + b)$ (C) $\frac{1}{2}(a + b)$ (D) \sqrt{ab} (E) $\sqrt{a^2 + b^2}$



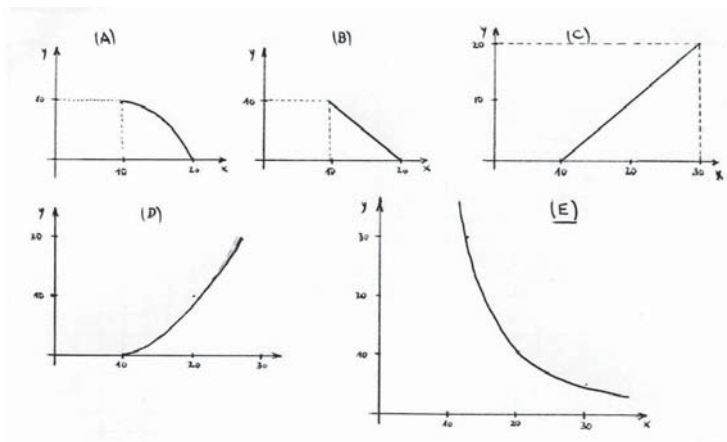
16. S RO.2 Let M be the set of all real numbers x for which the inequality $2^{4x} < 4^{2x}$ holds. Then $M =$

- (A) $(-\infty, 1)$ (B) $(0, 1)$ (C) $(-\infty, 1) \cup (1, \infty)$
 (D) $(0, \infty)$ (E) \mathbb{R}

17. S 3.41 A light bulb moves up vertically starting 10cm above a table. As shown in the picture a pencil 10cm long standing vertically on a table and 10cm apart from the light bulb casts a shadow on the table. Which is the graph of the length y of the shadow (in centimetres) as a function of the height x (in centimetres) of the bulb over the table?



- (A) (B) (C) (D) (E)

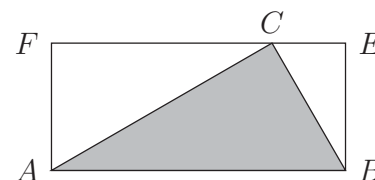


18. S 3.50 Two bottles of equal volume are both filled with a solution of water and acid. The ratios of the volume of water to acid are, respectively, $2 : 1$ and $4 : 1$. We pour all the contents of the two bottles into one big bottle. Then the ratio of water to acid in this bottle will be:

- (A) $3 : 1$ (B) $6 : 1$ (C) $11 : 4$ (D) $5 : 1$ (E) $8 : 1$

19. S 4.61 The diagram shows a rectangle $ABEF$ and a triangle ABC . We know that the angle ACF equals angle CBE . If $FC = 6$ and $CE = 2$ then the area of ABC is:

- (A) 12 (B) 16 (C) $8\sqrt{2}$
 (D) $8\sqrt{3}$ (E) another value

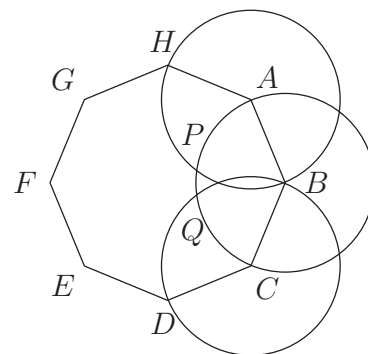


20. S 3.42 Charles says the truth every other day, otherwise he lies. Today he stated exactly four of the following sentences. Which one couldn't he have stated today?

- (A) I have a prime number of friends.
 (B) I have as many male friends as female.
 (C) 288 is divisible by 12.
 (D) I always say the truth.
 (E) Three of my friends are older than me.

27. S NP.5 In the figure, $ABCDEFGH$ is a regular octagon of side 1. Points P and Q are the intersections of circles with centres in A , B and C and radius 1. What is the size of $\angle APQ$?

- (A) $\frac{19}{24}\pi$ (B) $\frac{8}{11}\pi$ (C) $\frac{5}{8}\pi$ (D) $\frac{3}{4}\pi$ (E) $\frac{7}{9}\pi$

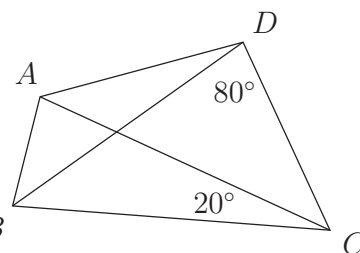


28. S 5.14 Start with a number, double it and then subtract 1. After applying this procedure 98 more times (starting each time from the previous result) you get $2^{100} + 1$. Which was the number you started with?

- (A) 1 (B) 2 (C) 4 (D) 6 (E) none of these

29. S 4.34 In the quadrilateral $ABCD$ the diagonal BD is the bisector of $\angle ABC$ and $AC = BC$. Given $\angle BDC = 80^\circ$ and $\angle ACB = 20^\circ$, $\angle BAD$ is equal to

- (A) 90° (B) 100° (C) 110° (D) 120° (E) 135°



30. S 5.18 Henry must travel from A to B and he plans to go at a certain speed. He would like to arrive earlier than planned and notes that travelling at a speed 5 km/h faster than planned he will arrive 5 hours earlier and travelling at a speed 10 km/h faster than planned he will arrive 8 hours earlier. His planned speed is

- (A) 10 km/h (B) 15 km/h (C) 20 km/h
(D) 25 km/h (E) impossible to determine